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REGULARISATION WITHIN THE SOURCE LOCALISATION METHOD SODIX

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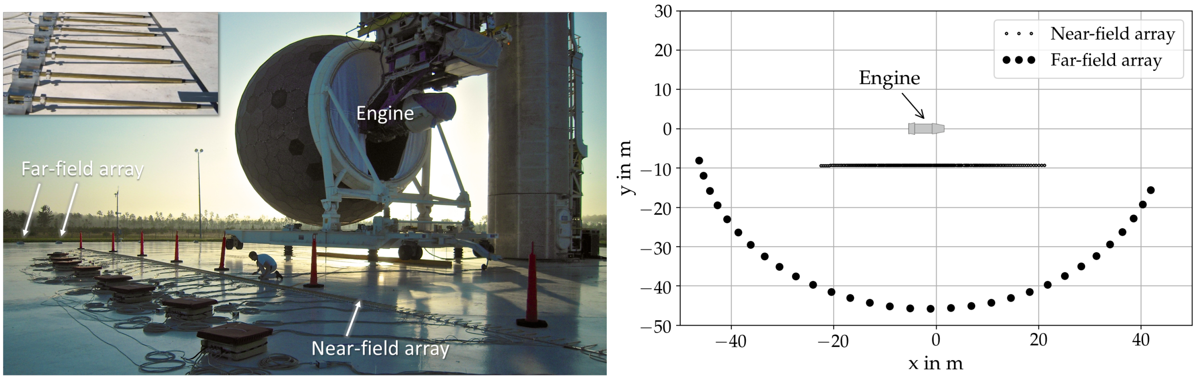
Abstract

The source localisation method SODIX is capable of determining the amplitudes and the directivities of sound sources. In comparison to other source localisation methods, the number of unknown source amplitudes to be determined is relatively high because the SODIX method models sound sources with individual amplitudes towards all array microphones. In order to prevent unstable solutions in the optimisation procedure, SODIX uses a regularisation that sets a constraint on the amplitudes changes of each source to nearby microphones. This paper compares this regularisation strategy with commonly used ℓ_1 and ℓ_2 regularisation techniques for two different numerically simulated testcases with non-directive and directive sound sources. The localisation results show that the currently used regularisation is beneficial for distributed sound sources as e.g. jet noise, while ℓ_1 and ℓ_2 regularisations effectively increase the dynamic range of the solution by minimising the influence of noise on the source amplitudes. However, these regularisation schemes do not yield accurate localisation results for distributed sound sources. Future work is required to combine the positive effects of all three regularisation techniques, e.g. by considering a linear combination of the regularisation schemes with accurate weighting.

1 INTRODUCTION

SODIX (SOurce DIrectivity modelling in cross-spectral matriX) is an acoustic source localisation method that is capable of determining the amplitude and the directivity of sound sources. A common application of SODIX is the localisation of sound sources of turbofan engines at static noise tests with a large microphone array near the engine, see figure 1. The method is able to quantify the contribution of individual source regions as e.g. the intake, the nozzle, and the jet, to the overall sound emission.



(a) Measurement setup at static engine tests [1] *(b) Microphone array positions*

Figure 1: Measurement setup with microphone arrays at a static free-field engine test stand. The dense near-field array consists of 248 microphones arranged in a line parallel to the engine axis with a polar resolution of 0.6°. The sparse circular array in the far-field consists of 31 microphones with a polar resolution of 5°.

In comparison to other source localisation methods, e.g. *Conventional Beamforming*, the number of unknown source amplitudes to be determined with SODIX is relatively high because the method models sound sources with individual amplitudes towards all array microphones. This can lead to ill-posed problems for large source grids or sparse microphone arrays. Therefore, the SODIX method uses a regularisation technique that can help to find stable solutions in the optimisation procedure and improve the localisation results. This regularisation strategy is physically motivated and relies on the local smoothness of the source directivities. It sets a constraint on the amplitude changes from a single source to different microphones which is a good assumption for broadband noise sources at static engine noise tests. However, this regularisation technique requires additional parameters that have to be known *a priori*, namely the number of neighbouring microphones to be included in the regularisation. A generalisation of this regularisation technique for different setups of sound sources (e.g. different engine speeds and frequencies in static engine noise tests) might not be trivial as shown in [2]. Additionally, the accurate weighting of the regularisation term against the model fit is not trivial either. The optimal value depends on the number of microphones in the array, the characteristics of the sound sources such as amplitude and directivity, and the overall sound pressure level measured by the microphones. The interaction of all these parameters complicates an accurate scaling of the regularisation against the source localisation.

Other regularisation techniques as ℓ_1 and ℓ_2 schemes are commonly used in the acoustic source localisation community, see e.g. [3–6]. These two approaches are also physically motivated and set a constraint on the sparsity (ℓ_1) and the energy (ℓ_2) of the solution vector. Both strategies do not rely on additional parameters and especially for the ℓ_2 regularisation, different techniques regarding the selection of the optimal scaling value have been developed [7, 8] that might also be beneficial for the SODIX method. However, it has not yet been studied whether these regularisation techniques are in general applicable to the source localisation with SODIX. Therefore, this paper creates a link between the currently used regularisation and the ℓ_1 and ℓ_2 regularisation schemes by studying the feasibility of applying the different regularisation terms to the source localisation with SODIX. The method is extended by the ℓ_1 and ℓ_2 regularisation schemes and their individual influence on the source localisation with SODIX is studied for synthesised sound sources with and without directivity. First, SODIX is applied individually for all three regularisation schemes to a set of monopole sources radiating in a 2D plane. Then, an additional study is carried out on a simple engine noise model that features directive source amplitudes in order to account for the capability of SODIX to resolve directive sound sources.

2 METHODOLOGY

2.1 Source localisation method SODIX

The SODIX method was developed by Michel and Funke [9–11] as an extension of the *Spectral Estimation Method (SEM)* by Blacodon and Élias [12, 13] and has mainly been applied to microphone measurements in static engine noise tests.

The method fits a model of the cross-spectral matrix to measured data from a microphone array. The SODIX model of the cross-spectral matrix consists of incoherent point sources D_{jm} with individual source amplitudes from all sources $j = 1 \dots J$ to all microphones $m = 1 \dots M$ of the array:

$$C_{mn}^{\text{mod}} = \sum_{j=1}^{J} g_{jm} D_{jm} D_{jn} g_{jn}^{*} \,, \tag{1}$$

with g_{jm} representing the steering vector for the source j to the microphone m. Throughout this paper, free-field propagation is assumed with $g_{jm} = \frac{1}{r_{jm}} e^{ikr_{jm}}$, where r_{jm} is the distance from source j to the microphone m, and k is the wave number. The directive source amplitudes D_{jm} are determined by a least-squares fit between the measured and the modelled cross-spectral matrix:

$$F(D) = \sum_{m,n=1}^{M} \left| C_{mn} - \sum_{j=1}^{J} g_{jm} D_{jm} D_{jn} g_{jn}^{*} \right|^{2} \,. \tag{2}$$

This leads to an optimisation problem for the unknown source directivities D_{jm} that is solved indirectly with an iterative minimisation procedure based on conjugate gradients, see also [14].

2.2 Regularisation schemes

In comparison to other source localisation methods, the number of unknown source amplitudes to be determined in the SODIX model is relatively high: $D \in \mathbb{R}_{+}^{J \times M}$ where J is the source number and M is the microphone count. The problem formulation in equation 2 might be ill-posed for large source grids when the number of microphones is small compared to the number of sources. In this case, additional regularisation techniques are required in the optimisation process of equation 2 in order to find stable solutions for the unknown source directivities. A generalisation of the SODIX objective function, including regularisation, can be written as:

$$F(D) = \sum_{m,n=1}^{M} \left| C_{mn} - \sum_{j=1}^{J} g_{jm} D_{jm} D_{jn} g_{jn}^{*} \right|^2 + \sigma R(D) \ . \tag{3}$$

In this equation, R is the underlying regularisation scheme and σ is a weighting factor that controls the regularisation against the model fit, namely the modelling of the cross-spectral matrix with directive sound sources. The regularisation is directly included in the iterative minimisation process of the objective function. The partial derivatives of model fit and regularisation scheme are calculated at each iteration of the conjugate gradient method which can increase the computational time for complicated regularisation schemes.

So far, the SODIX method uses a regularisation technique that is physically motivated and is based on the local smoothness of the source directivities. This regularisation scheme sets a constraint on the amplitude changes from a single source to neighbouring microphones:

$$R_{\mathrm{S}}(D) = \sum_{j=1}^{J} \sum_{m=1}^{M} \sum_{l=1}^{L(m)} \alpha_l \left(D_{jm} - D_{j,\Lambda(l)} \right)^2 \ . \tag{4}$$

In this strategy, the source directivity D_{jm} for a particular source j in direction to a microphone m is determined by including the source amplitudes of that same source to neighbouring microphones. Large deviations between amplitudes of the source to neighbouring microphones are then penalised within the optimisation procedure. In this context, the regularisation scheme R_{S} can be seen as a high-pass operator that will work as a low-pass operator in the inversion process which prevents unphysical large local variations in the directivity of the sources. In equation 4, L stands for the number of neighbouring microphones that are taken into account in the regularisation. This additional parameter has to be set *a priori* and might vary for different configurations of sound sources and microphone arrays. The coefficient α_l represents a linear weighting of the considered directivities with the corresponding distances r: $\alpha_l = 1 - r(l)/r_m$.

Other regularisation strategies are commonly used in the source localisation community. Most of all, ℓ_1 and ℓ_2 regularisation have been applied to the acoustic inverse problem, see e.g. [3–6]. Both schemes are also physically motivated and set a constraint on the sparsity (ℓ_1) and the energy (ℓ_2) of the solution. The schemes for ℓ_1 and ℓ_2 regularisation are respectively given by equation 5 and 6 and do not rely on further parameters.

$$\ell_1 : \ R_1(D) = \|D_{jm}\|_1 = \sum_{j,m} |D_{jm}| \ , \tag{5}$$

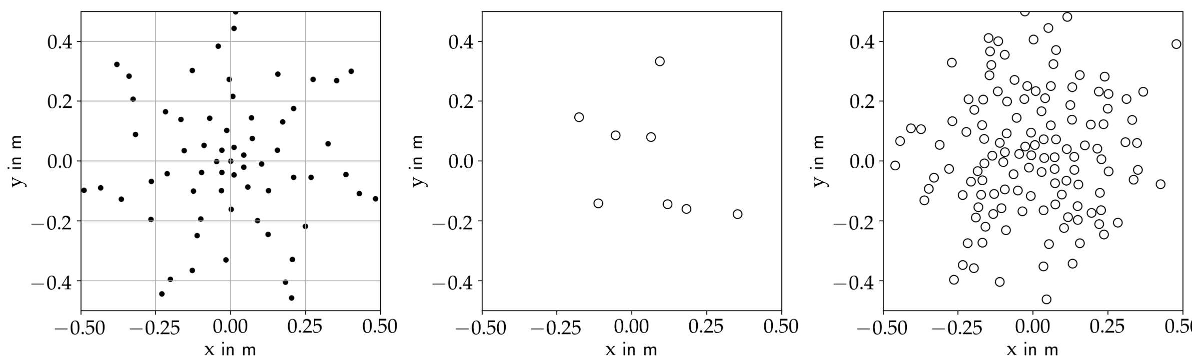
$$\ell_2 : \ R_2(D) = \|D_{jm}\|_2^2 = \sum_{j,m} D_{jm}^2 \ . \tag{6}$$

Linear combinations of all three proposed regularisation schemes R_{S}, R_1 and R_2 might also be applicable, but are out of the scope of this study. It should also be noted that the three regularisation techniques can require different scaling values σ_{S}, σ_1 and σ_2. However, this paper only focusses on the general feasibility of each regularisation when applied to the source localisation with SODIX. The accurate selection of the scaling parameter σ is the subject of ongoing research.

3 APPLICATION TO NON-DIRECTIVE SOUND SOURCES

3.1 Simulated sound sources and microphone setup

In order to investigate the influence of the different regularisation schemes within the source localisation method SODIX, a study with non-directive sound sources is presented that simulates 8 respectively 128 equally strong monopoles radiating in a 2D plane. Each source has an amplitude of 1 Pa or 94 dB at a distance of one metre. The simulation frequency is approximately 10 kHz and the sources are spatially separated by at least one wavelength. The cross-spectral matrix is synthesised for a spiral array with only 64 microphones at a distance of one metre from the source plane. Additional noise with a signal-to-noise ratio of 30 dB was incoherently added to the main diagonal of the simulated cross-spectral matrix. Figure 2 indicates the spiral array and the simulated source positions for both setups.



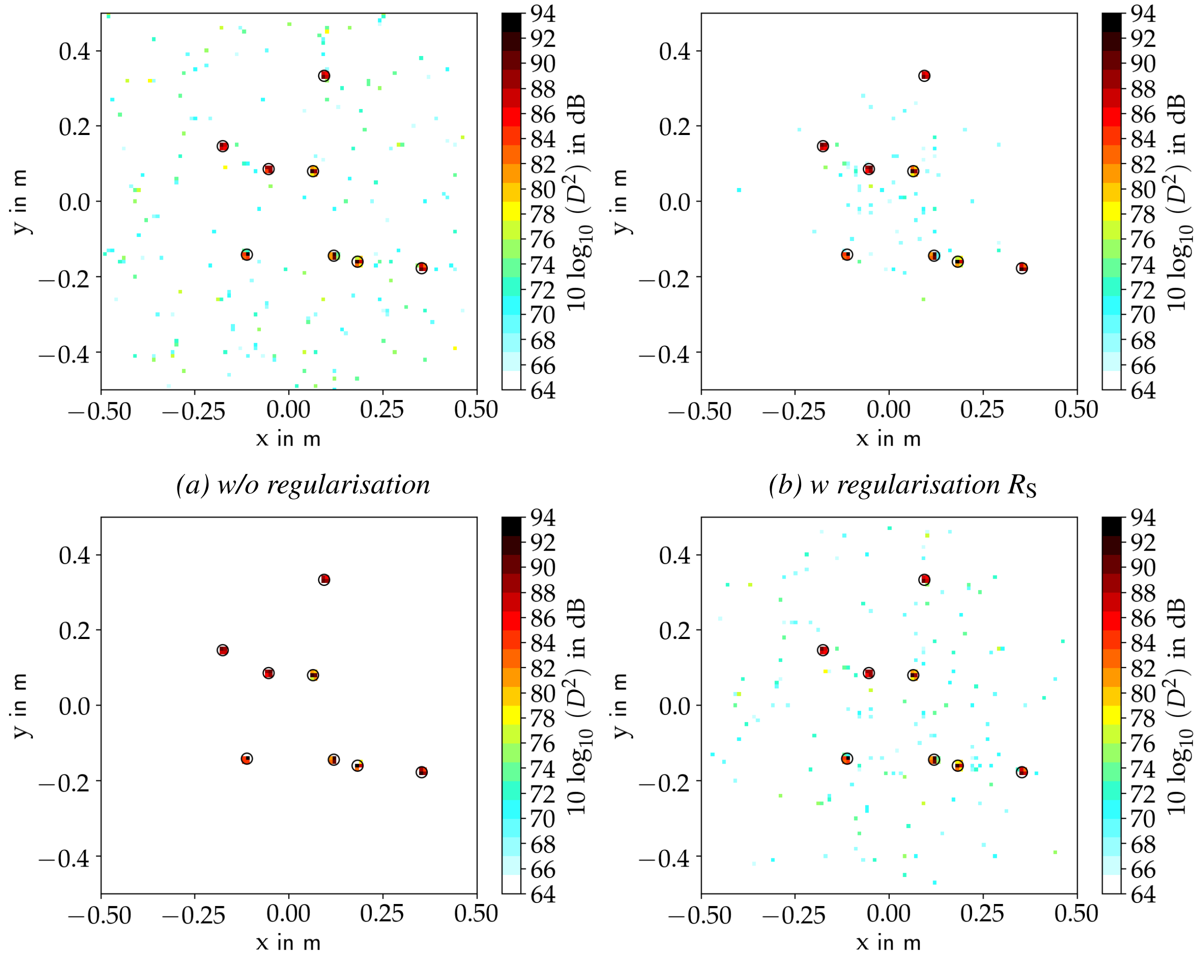
(a) Spiral array from [15] *(b) Positions of 8 monopoles* *(c) Positions of 128 monopoles*

Figure 2: Spiral array with 64 microphones (a) and source positions of 8 (b) and 128 (c) equally strong monopoles radiating in a 2D plane that is at a distance of one metre from the microphone array plane.

For the source localisation, a rectangular grid is used that consists of 10404 points with an equidistant spacing of 0.16 wavelengths. In this case, the number of unknown directional source amplitudes to be modelled by SODIX is 665856, and therefore much higher than the number of independent values in the cross-spectral matrix which is only 2080.

3.2 Results derived with SODIX for different regularisation schemes

Figure 3 and 4 show the source localisation results calculated with SODIX for both setups. The source maps present the source localisation results without any regularisation (top left), followed by those with the regularisation schemes R_S, R_1, and R_2 applied individually. The SODIX results have been averaged over the directivity ($\overline{D_j^2} = \sum_m D_{jm}^2/M$) in order to present 2D source maps that can be compared to those of other source localisation methods as Conventional Beamforming and Functional Beamforming.



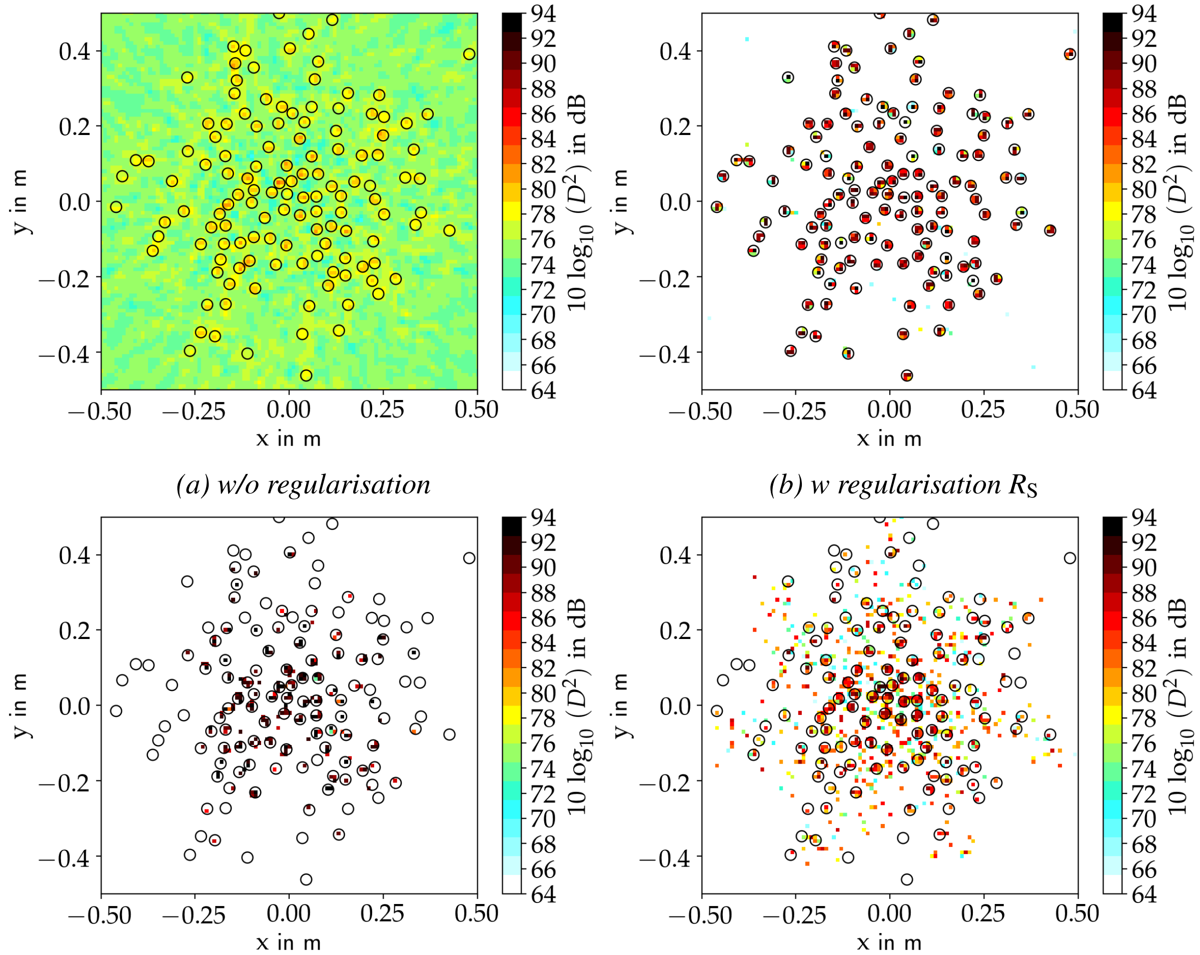
(a) w/o regularisation

(b) w regularisation R_S

(c) w regularisation R_1

(d) w regularisation R_2

Figure 3: Source localisation results derived with SODIX for a simulation of 8 monopole sources perceived by a spiral array with 64 microphones. The maps show the source amplitudes determined with SODIX without regularisation and with regularisation schemes R_S, R_1, and R_2 applied individually.



(a) w/o regularisation *(b) w regularisation R_S*

(c) w regularisation R_1 *(d) w regularisation R_2*

Figure 4: Source localisation results derived with SODIX for a simulation of 128 monopole sources perceived by a spiral array with 64 microphones. The maps show the source amplitudes determined with SODIX without regularisation and with regularisation schemes R_S, R_1, and R_2 applied individually.

In figure 3, SODIX is able to localise all 8 sources very well, even when no regularisation is applied. In this case, the number of directive sources that have a significant amplitude is 512 ($\hat{J} \cdot M = 8 \cdot 64$) and hence lower than the number of 2080 independent values in the cross-spectral matrix. The problem is therefore well-posed and no further regularisation is required to improve the localisation results. The application of different regularisation schemes has a minor effect on the distribution of spurious secondary sources that show up on positions different from those of the actual simulated sources. The best source localisation results are archived for the R_1 scheme which is known to work well for sparse solutions with only a few dominant sound sources. The regularisation schemes R_S and R_2 slightly improve the dynamic range of the source solution by decreasing secondary sound sources, however this effect is rather low.

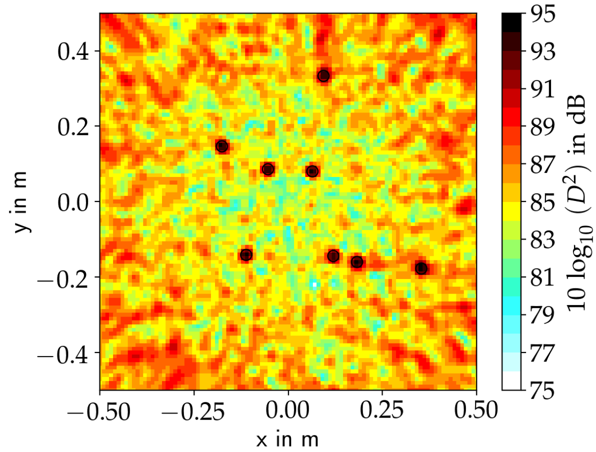
For the simulated testcase with 128 monopole sources in figure 4, the localisation without regularisation leads to a source map with rather low dynamic range. The source positions indicated by the black circles are not accurately detected and the integrated source amplitudes do not match the simulated source amplitudes. In this case, the problem is ill-posed because the number of significant source amplitudes to be modelled is 8192 ($\hat{J} \cdot M = 128 \cdot 64$) and much larger than the 2080 independent values in the cross-spectral matrix. Therefore, regularisation is required to effectively improve the source localisation results. Applying the regularisation scheme R_S increases the dynamic range of the solution derived with SODIX. All synthesised point sources can be separated well and the source amplitudes are in good agreement with the simulated amplitudes of 94 dB. The regularisation schemes R_1 and R_2 also improve the dynamic range and the source separation in comparison to the results without any regularisation. However, in contrast to the currently used regularisation R_S, both ℓ_1 and ℓ_2 regularisation schemes are not capable to localise all sources within a dynamic range of 30 dB. Especially sound sources remote from the clustered centre are not sufficiently detected. It is concluded that the clustering of a large number of sound sources is a complication for the R_1 and R_2 regularisation strategies when used in SODIX.

3.3 Results derived with Conventional Beamforming and Functional Beamforming

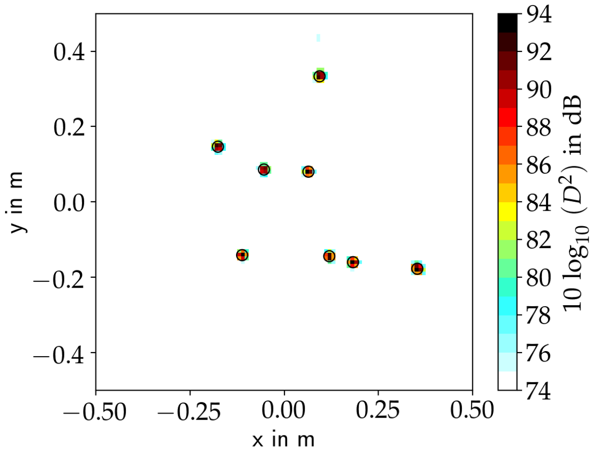
To place the SODIX method in a broader context of acoustic source localisation methods, results are presented in this section that were derived with Conventional Beamforming and Functional Beamforming, see also [16, 17] for the methodology.

Figure 5 shows the source localisation results derived with Conventional Beamforming on the left-hand side and with Functional Beamforming on the right-hand side. For the simulation with 8 monopoles, the source map derived with Conventional Beamforming has a relatively low dynamic range of 10 dB. Nevertheless, all 8 sources are accurately detected at the simulated positions. Remember that the SODIX source map in figure 3 has shown a much higher dynamic range for the same setup. It is well known that indirect methods as SODIX can provide solutions with larger dynamic range than direct methods as beamforming, see e.g. [13]. The low dynamic range of the beamforming map results from the convolution of the source distribution with the point spread function of the array. In the case of 8 monopole sources, Functional Beamforming clearly improves the dynamic range of the solution by effectively reducing sidelobes even for low values of the functional exponent $\nu = 5$.

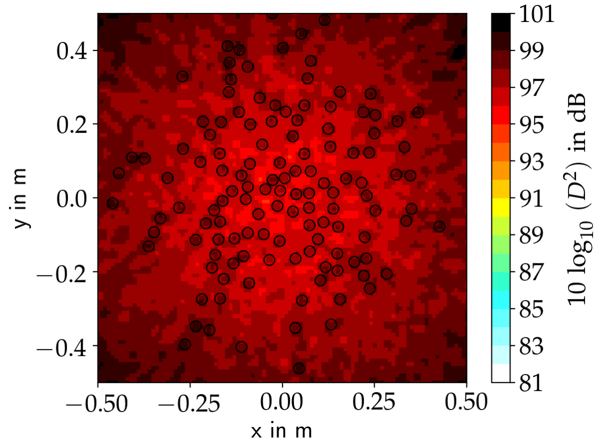
For the simulation with 128 monopole sources, neither Conventional Beamforming nor Functional Beamforming yield accurate localisation results. In this case, a deconvolution method as DAMAS might be a better choice.



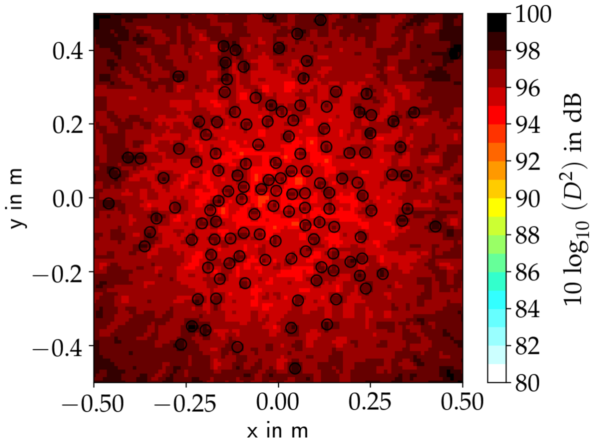
(a) Conventional Beamforming



(b) Functional Beamforming ($\nu = 5$)



(c) Conventional Beamforming



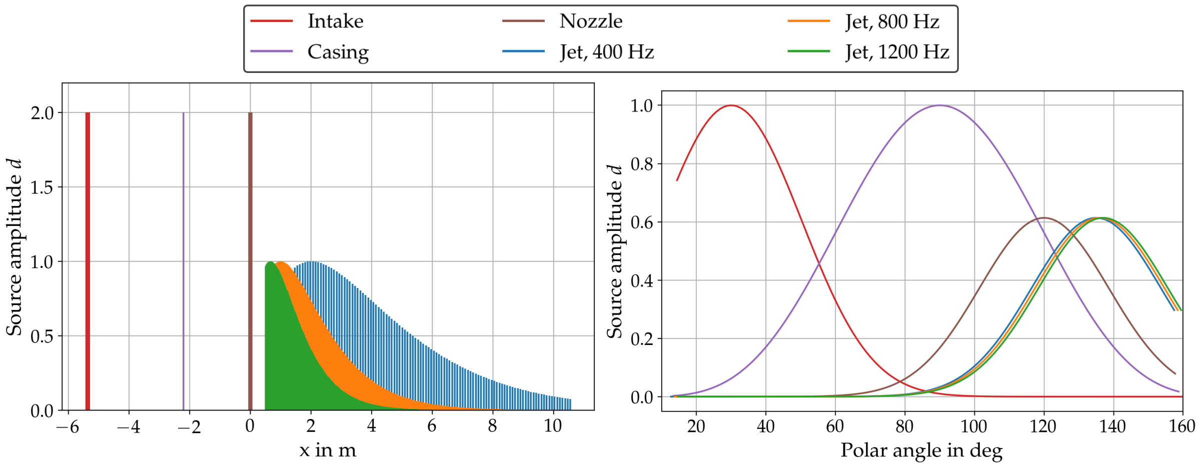
(d) Functional Beamforming ($\nu = 100$)

Figure 5: Source localisation results derived with Conventional Beamforming (left column) and Functional Beamforming (right column) for a simulation of 8 respectively 128 monopole sources perceived by a spiral array with 64 microphones.

4 APPLICATION TO DIRECTIVE SOUND SOURCES

4.1 Simulated sound sources and microphone setup

In this section, the SODIX method with different regularisation schemes is applied to a simple engine noise model. The simulation includes directive sound sources in order to demonstrate the special abilities of SODIX. The testcase is a very simplified model for the sound emission of a turbofan engine and its only purpose is to demonstrate the capabilities of SODIX rather than to provide a model of the sound emission of real aero-engines. The benchmark case was first presented in [2] to study the application of SODIX to sparse microphone arrays in the far-field and has now been adopted to a microphone array that is aligned in parallel with the engine axis. Compared to static noise tests, the linear array used in this simulation is relatively sparse with only 50 microphones. Figure 6 depicts the simulated source amplitudes and the corresponding directivities for the source regions intake, casing, nozzle, and jet.



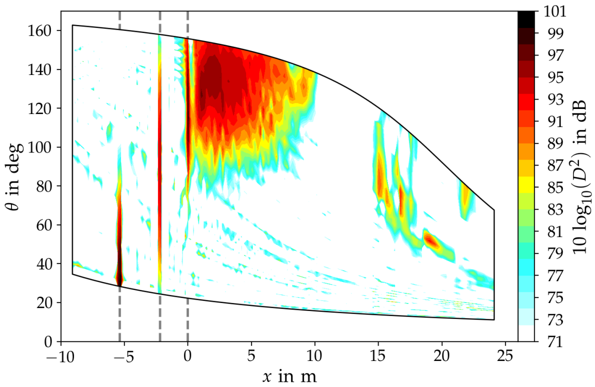
(a) Source amplitudes against source position *(b) Source directivities against polar angle*

Figure 6: Source amplitudes and source directivities for a simple engine noise model based on point sources on the engine axis. The intake and the nozzle are modelled by three incoherent point sources that are separated by one twentieth of the wavelength. The jet noise is modelled by a line of point sources with frequency dependent position of the maximum amplitude. [2]

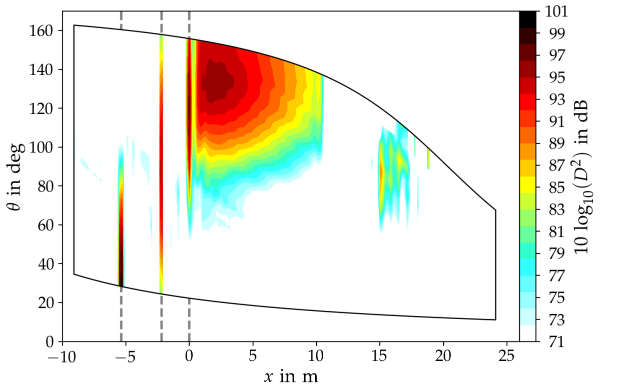
The simulated point sources are located on the engine axis. The position of the engine intake is at $x \approx -5$ m and the nozzle exit is at $x = 0$ m. The sound fields from the intake and the nozzle are each described by three individual point sources that are separated one twentieth of the wavelength respectively. The jet is modelled by an analytical source distribution on the engine axis with a frequency dependent source position of the maximum amplitude as described by Glegg [18]. Additionally, a single point source with rather small amplitude is simulated on the engine casing between intake and nozzle. The directivity of the intake sources is synthesised by a sinusoidal function with maximum intensity at polar angles of 30°. The directivity of the nozzle sources is given by a Gaussian distribution that radiates with maximum intensity at polar angles of 120°. The jet directivity is assumed to be frequency independent with a maximum level at 135°. The directivity of the additional source on the casing is described by a sinusoidal function with maximum level at polar angles of 90°. Additional noise with a relatively low signal-to-noise ratio of 10 dB was incoherently added to the main diagonal of the simulated cross-spectral matrix.

4.2 Results derived with SODIX for different regularisation schemes

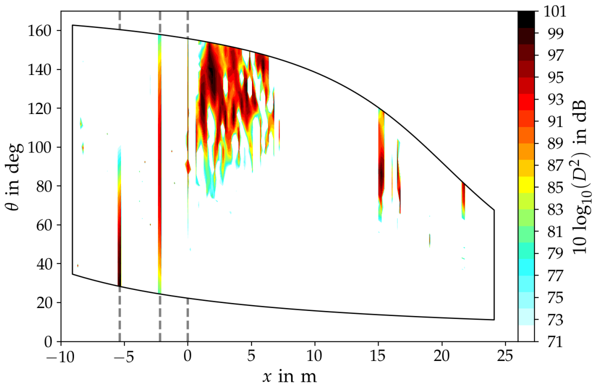
Figure 7 shows a qualitative comparison of the source localisation results derived with SODIX for different regularisation schemes to those without any regularisation applied (top left). In contrast to the previous section, these source maps show the directive source amplitudes against the axial position of the sources on the horizontal axis and the emission angle on the vertical axis. The grey, dashed lines indicate the simulated source positions of intake, casing, and nozzle. Each map presents a dynamic range of 30 dB.



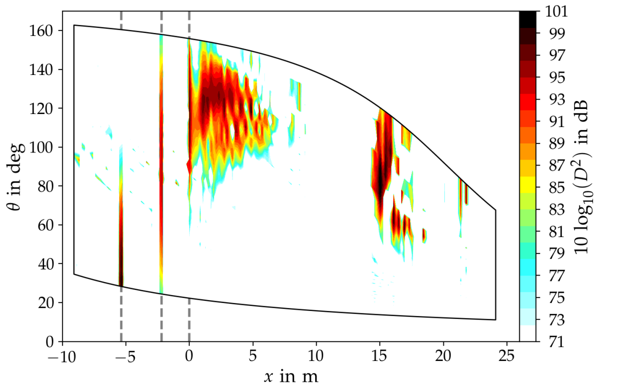
(a) w/o regularisation



(b) w regularisation R_S



(c) w regularisation R_1



(d) w regularisation R_2

Figure 7: Source localisation results derived with SODIX for a directive engine noise model at 400 Hz. The maps show the source amplitudes determined with SODIX without regularisation and with regularisation schemes R_S, R_1, and R_2 applied individually.

Figure 7 shows that the SODIX method without any regularisation applied is able to localise the sound sources at their simulated positions. However, the dynamic range of the solution is lower compared to the cases when a regularisation is applied. The modelled source distribution in the jet area is not as continuous as the simulated distribution due to the low signal-to-noise ratio. In addition, spurious sound sources appear in the area downstream of the jet. These secondary sources are not related to the simulated sources and similar spurious sources have been observed in the SODIX solutions for a sparse far-field array in a static noise test with a real engine [2].

The application of the regularisation scheme R_S improves the localisation with SODIX. The dynamic range of the solution seems to increase because spurious secondary sources are reduced. In addition, the jet sources are now modelled by a continuous distribution of point sources which is a clear benefit of this regularisation technique.

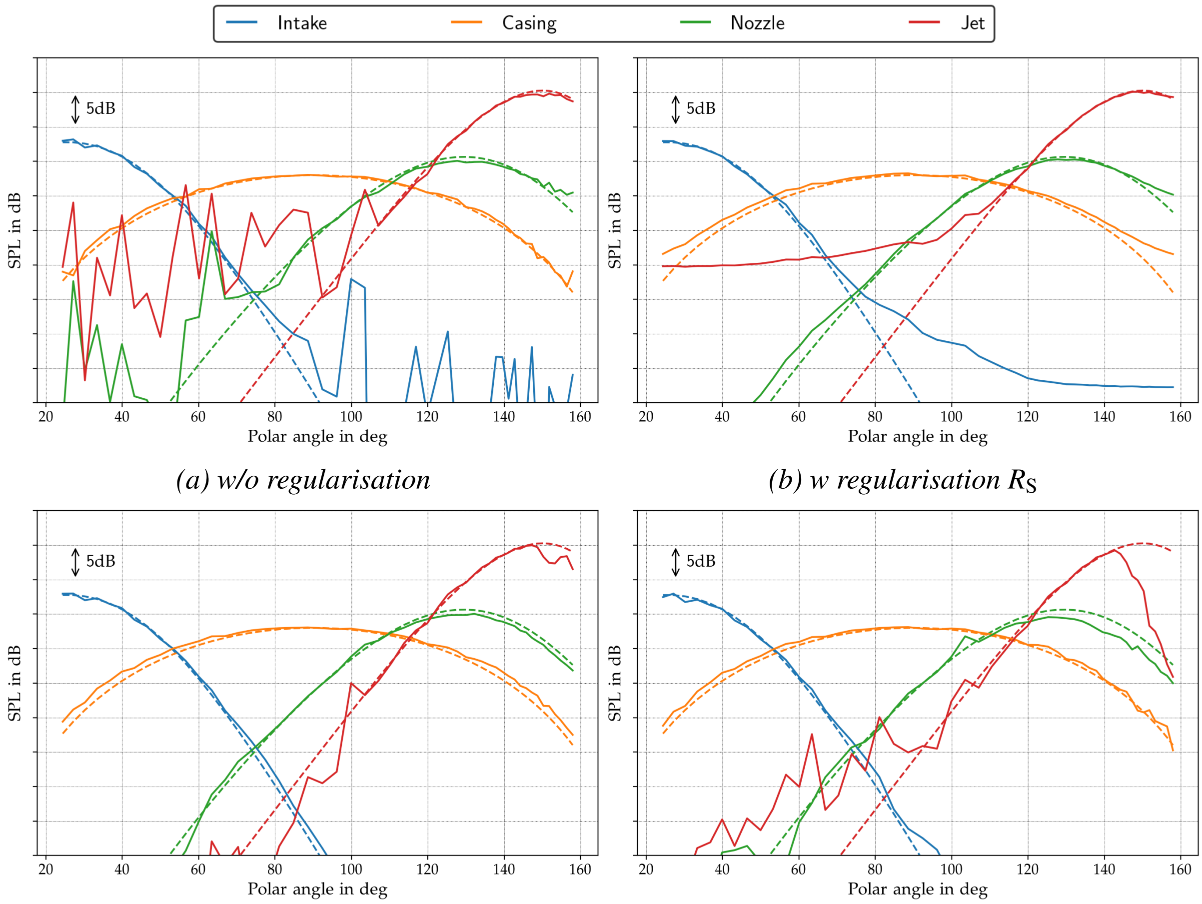
The ℓ_1 and ℓ_2 regularisation schemes improve the dynamic range of the solution by reducing secondary sources that appear due to the low signal-to-noise ratio. Both schemes work well for the relatively sparse sources at intake, casing, and nozzle. Especially the R_1 scheme helps to improve the source separation between the nozzle and the jet. However, both regularisation techniques seem to fail in the jet area where SODIX models a relatively distorted source distribution with large local variations of the source amplitude. This phenomenon is consistent with the observation from the simulation of monopole sources that both ℓ_1 and ℓ_2 regularisations do not work well for clustered (or spatially distributed) sources. The spurious artefacts in the area downstream of the jet are also amplified by both regularisation schemes.

The quality of the source localisation for different regularisation schemes can be evaluated by comparing the individual directivities of the source regions intake, casing, nozzle, and jet to those of the simulated sources. To do this, the directive source amplitudes were integrated for different source regions $\mathbb{S}$ that match the simulated source positions: $\overline{D_m^2} = \sum_{j \in \mathbb{S}} D_{jm}^2$. Figure 8 shows a comparison of the integrated source directivities for different regularisation schemes against the simulated sources.

The simulated source directivities are relatively well modelled with SODIX, even when no regularisation is applied. In the dynamic range of approximately 25 dB, the deviations between modelled and simulated directivities are within 1 dB over a wide polar range. However, larger local variations of the source directivity can be observed for amplitudes below this dynamic range that result from the simulated signal-to-noise ratio.

The regularisation R_S improves the localisation with SODIX by penalising these large local variations in the optimisation procedure. The scheme works as a low-pass operator on the directivity of the sources which reduces the unreliable peaks from the unregularised solution. However, this regularisation technique only removes the large amplitude variations, especially in the directivity of the jet noise, rather than improving the overall dynamic range of the source solution. A secondary effect of the R_S regularisation is visible for the sources at casing and nozzle at high emission angles. The source directivities become unreliable uniform because the selected scaling value σ_S is too large. On the other hand, the same scaling value yields good results for the source directivity of intake and jet which shows that further modifications of the regularisation scheme R_S might be required to find the optimal solution.

Both R_1 and R_2 schemes effectively increase the dynamic range of the solution by reducing the influence of noise on the source amplitudes. Even source directivities that are 40 dB below the maximum amplitude are well captured, see e.g. the intake source. However, as described for the source maps, both schemes are not able to accurately model the continuous source distribution of the jet. This drawback could be resolved by considering a linear combination of the regularisation schemes R_S, R_1, and R_2 which is the subject of ongoing research.



(a) w/o regularisation *(b) w regularisation R_S*

(c) w regularisation R_1 *(d) w regularisation R_2*

Figure 8: Comparison of simulated and localised source directivities with SODIX for a directive engine noise model at 400 Hz. The results were derived without regularisation and with regularisation schemes R_S, R_1, and R_2 applied individually.

5 CONCLUSIONS AND OUTLOOK

The feasibility of applying different regularisation schemes within the source localisation method SODIX has been studied. Besides the currently used regularisation technique that sets a constraint on large variations in the directivity of the sound sources, the SODIX method was extended by applying the commonly used ℓ_1 and ℓ_2 regularisation schemes. Their impact on the source localisation was demonstrated using synthesised sound sources with and without directivity. First, SODIX was applied to a set of monopole sources radiating in a 2D plane. Second, an additional study was carried out on a simple engine noise model with directive sound sources.

The localisation results have shown that all three regularisation schemes are applicable to SODIX, but their benefit depends on the distribution of the sound sources. The currently used regularisation scheme R_S works as a low-pass operator and reduces large local variations of the source directivity. However, the scheme does not significantly improve the dynamic range of the solution. In contrast to that, the ℓ_1 and ℓ_2 regularisation schemes clearly improve the dynamic range of the source solution by minimising the influence of noise on the modelled source amplitudes. These schemes work only well for sparsely populated source regions and fail for distributed sound sources as jet noise. In the case of distributed or clustered sources, the regularisation scheme R_S is beneficial because it helps SODIX to model a continuous source distribution and directivity.

Further research is required to combine the positive effects of all three regularisation schemes, e.g. by a linear combination. The selection of the optimal scaling value of the regularisation is also the subject of ongoing research.

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