



# USING MIRROR IMAGE SOURCES TO INCREASE PERFORMANCE OF BEAMFORMING IN CLOSED ENCLOSURES

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## ABSTRACT

Performance of the basic delay and sum beamforming is influenced by the reflections of sound from surfaces in the environment. In closed spaces this effect is even more significant, where signals from the high number of close secondary mirror image sources interfere with the primary signal. However, one can also make use of the presence of these image sources. If the geometry of the surrounding surfaces is known, one can also focus on the closest secondary image sources, create secondary sound maps (images) and combine them with the primary ones. The advantage of focusing on the secondary sources is that they are seen under independent direction characteristics, which allows the elimination of real interferences, obstructing objects.

In this paper we present the results of our investigations of employing room acoustical considerations to increase efficiency and performance of a random-placed microphone array used in a closed enclosure. A simple case of a shoebox shaped room is used for the simulations. The effect, advantages and limits of using sound maps created by second and higher order image sources is discussed.

## 1 INTRODUCTION

The application range of beamforming with microphone arrays is dominated by free-field cases [5]. Although there exist indoor, closed space applications as well, for example in building acoustics, their number is minimal [2, 3]. The reason for this is that Green's function of a closed enclosure is much more complex than that of the free-field. In this paper we examine the effect of the bounding surfaces of the enclosure to the performance of basic delay-and-sum algorithm.

If we place a sound source in a closed enclosure, its sound field will consist of two distinct parts: the direct sound field and the reverberant one, the latter is generated by the reflections

from the bounding surfaces. The reflection paths can be handled as primary paths from the mirror image of the primary source.

In this paper two simple beamforming approaches are investigated. The first one is a rather physical one, where the focusing is achieved by compensating the delays of the primary transfer paths from the focus point to the each member of the microphone array. The second one is a more mathematical approach: the steering vector is selected to yield an optimal point spread function over the entire beamforming plane.

We show that the beamforming algorithms optimised for free-field applications fail in closed spaces. We present the modified versions of the investigated two algorithms, which employ information about the first and second order reflection paths in order to adopt to closed space conditions.

## 2 DELAY AND SUM BEAMFORMING

Let  $p_m$  denote the pressure at the  $m$ -th microphone of a microphone array consisting of  $M$  elements. The image point  $b_f$  is composed from the microphone pressures by weighting them with a weighting vector  $\mathbf{w}_f$  as follows [1]:

$$b_f = \sum_{m=1}^M w_{fm} p_m, \quad \mathbf{b} = \mathbf{W}\mathbf{p} \quad (1)$$

The microphone pressures are the response of the sound field to a set of excitation sources  $x_s$ ,  $s = 1, \dots, S$ . Let the propagation path between the  $s$ -th source and the  $m$ -th microphone be described by the propagation amplification  $g_{ms}$ , resulting in

$$p_m = \sum_{s=1}^S g_{ms} x_s, \quad \mathbf{p} = \mathbf{G}\mathbf{x} \quad (2)$$

We assume that the  $m$ -th microphone is located at  $\mathbf{r}_m$ , while the  $s$ -th source is located at  $\mathbf{r}_s$ . The resulting point spread function of the microphone array is defined by

$$\mathbf{C} = \mathbf{W}\mathbf{G}, \quad \mathbf{b} = \mathbf{C}\mathbf{x} \quad (3)$$

Optimally, the point spread function is close to identity, meaning that the image approximately represents the real sources ( $\mathbf{b} \approx \mathbf{x}$ ).

## 3 THE INITIAL MODEL

Using the straightforward approach, the weighting factors are chosen such that they compensate the propagation delay and attenuation exactly for each focus point. This leads to the following formulation [4]:

$$w_{fm} = \frac{1}{M g_{fm}} \quad (4)$$

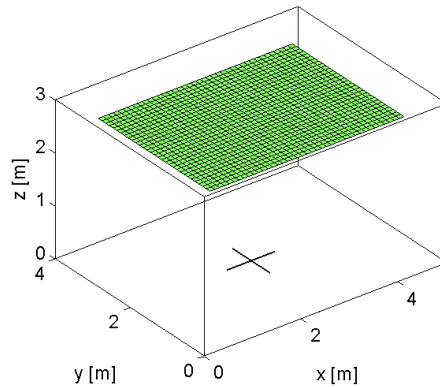


Figure 1: Basic simulation setup with a cross shaped array of 48 microphones

where, if one assumes omnidirectional point sources, microphones and free field propagation,  $g_{fm}$  is the symmetrical full space Green's function:

$$g_{fm} = g_{mf} = g(\mathbf{r}_f - \mathbf{r}_m) = \frac{e^{-ik|\mathbf{r}_f - \mathbf{r}_m|}}{4\pi|\mathbf{r}_f - \mathbf{r}_m|} \quad (5)$$

Combination of equations (1 -4) yields

$$b_f = \sum_{m=1}^M \frac{1}{Mg_{fm}} \sum_{s=1}^S g_{ms} x_s = \sum_{s=1}^S \left( \frac{1}{M} \sum_{m=1}^M \frac{g_{ms}}{g_{fm}} \right) x_s \quad (6)$$

where the resulting point spread matrix is given by

$$c_{fs} = \frac{1}{M} \sum_{m=1}^M \frac{g_{ms}}{g_{fm}} \quad (7)$$

### 3.1 Simulation setup

In the following, the basic properties of the initial model are presented using two basic microphone array shapes. The first is a cross shaped array of 48 microphones, with a microphone spacing of 5 cm, giving an array size of  $1.15 \times 1.15$  m. The second array is a randomly spaced plane microphone set, where 24 microphones are distributed along the same  $1.15 \times 1.15$  m area. The image plane is parallel to the microphone array, and is located at a distance of 2.7 m from the array. The image resolution is also 5 cm, the image size is  $4 \times 3$  m. The setup is plotted in Figure 1 for the case of the cross shaped array.

Figure 2. displays the point spread functions of the two arrays at two different frequencies:  $k_{\text{low}} = 25$  rad/m and  $k_{\text{high}} = 50$  rad/m. The Nyquist frequency of the array is at  $k_{\text{Nyq}} = 63$  rad/m.

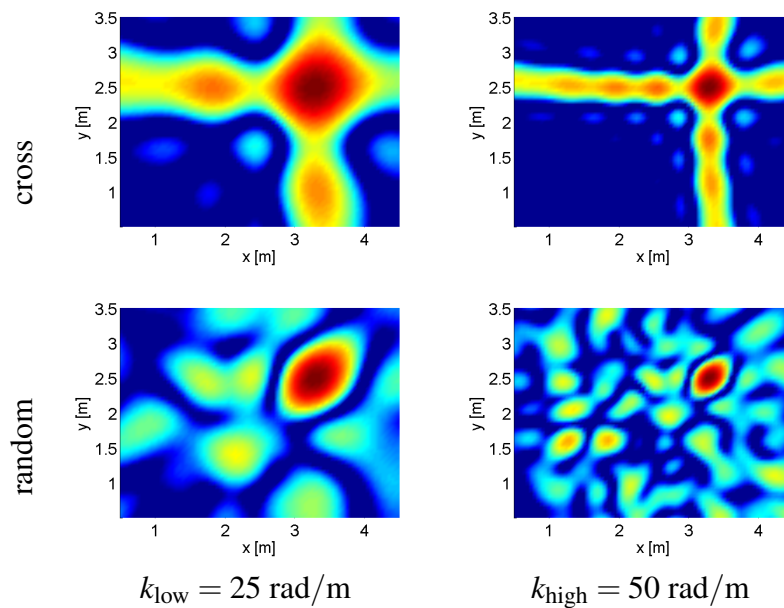


Figure 2: Free field point spread functions of a cross shaped and randomly spaced microphone array at two different frequencies

### 3.2 The initial model in closed space

If the microphone array is located in an enclosure with reflecting walls, equation (5) no longer holds. We introduce the notation  $g_{ms}^\alpha$  for the propagation amplification in an enclosure so that reflections up to the order  $\alpha$  are taken into account. With this notation, we generalise the point spread function of the microphone array as

$$\left[ \mathbf{C}_{\alpha,\beta}^P \right]_{fs} = \frac{1}{M} \sum_{m=1}^M \frac{g_{ms}^\alpha}{g_{fm}^\beta} \quad (8)$$

With this definition we assume that the reflection paths up to order  $\beta$  are taken into account by the weighting factors, while reflections up to the order  $\alpha$  are present in the sound field.

In the following we present the effect of reflections on the point spread functions of the arrays. In the simulation model, only the walls perpendicular to the image plane are considered as reflecting. The amplitude reflection coefficient of the walls is set to  $r = 0.9$ , typical value for unfurnished empty rooms.

Figures 3-4 display the resulting point spread functions  $C_{\alpha,0}^P$ , where only the primary paths are accounted for by the weighting factors. The results clearly show that the presence of already a few (4-12) reflections totally blurs the PSFs. It is worth mentioning that the PSF of the random array seems to be more sensitive to the increasing number of reflections than that of the cross-shaped array.

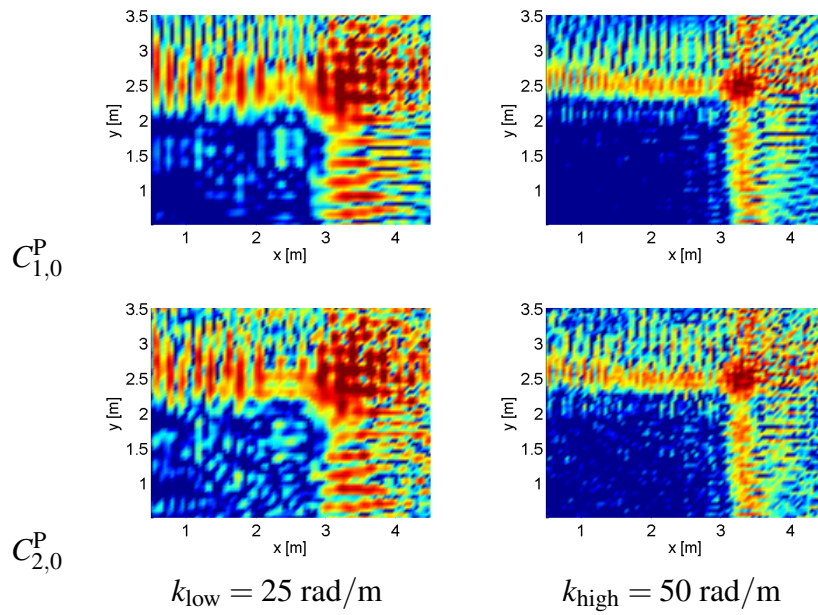


Figure 3: Point spread functions  $C_{\alpha,0}^P$  of the cross-shaped microphone array

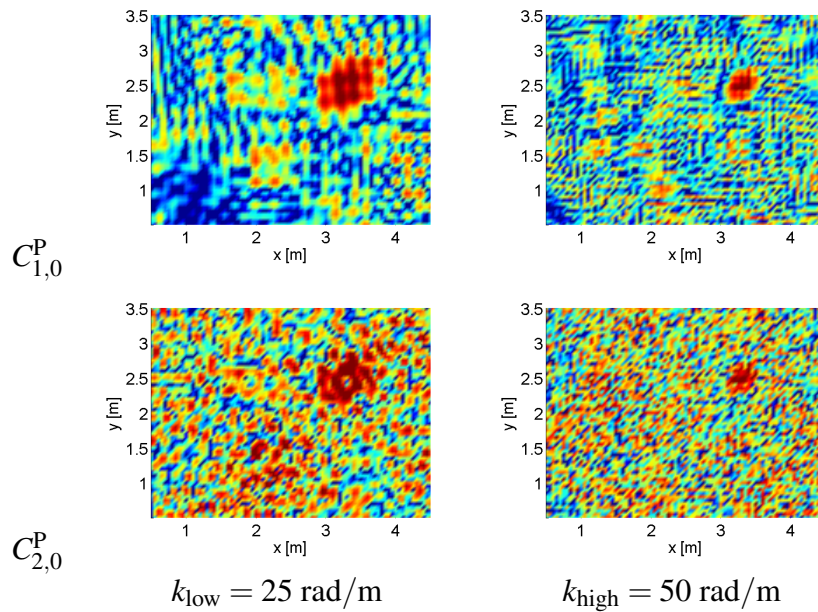


Figure 4: Point spread functions  $C_{\alpha,0}^P$  of the random microphone array

#### 4 MIMO MODEL IN OPEN AND CLOSED SPACES

For the case of the initial model, the weighting factors are determined in order to achieve a unit PSF at the focus point, regardless of the other PSF values. The MIMO model (Multiple Input Multiple Output) aims for a global optimum of the PSF by computing the pseudo inverse of

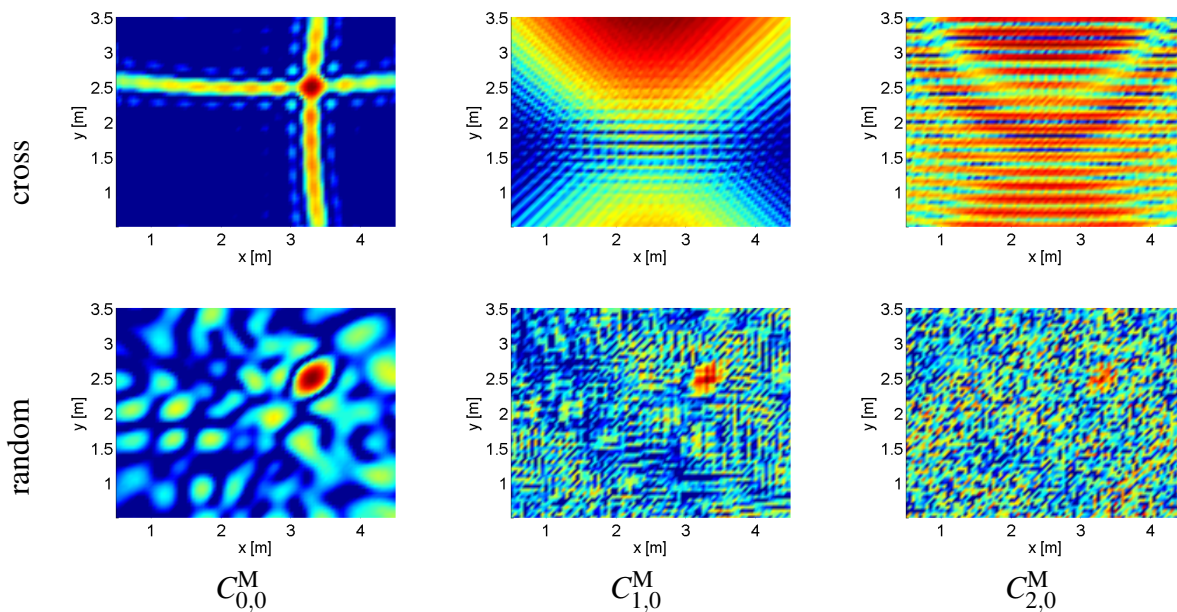


Figure 5: Point spread function  $C_{\alpha,0}^M$  of the two microphone arrays at the higher frequency case

the propagation matrix  $\mathbf{G}$ , and applying the pseudo inverse as the optimal weighting matrix:  $\mathbf{W}_{\text{opt}} = \mathbf{G}^+$ . The resulting point spread function is

$$\mathbf{C}_{\alpha,\beta}^M = \mathbf{G}_{\beta}^+ \mathbf{G}_{\alpha} \quad (9)$$

where again,  $\alpha$  and  $\beta$  denote the maximal order of reflections, accounted for by the propagation matrices.

The accuracy of the point spread matrix is limited by the number of microphones  $M$ . Obviously, this value is the maximum possible rank of the propagation matrix  $\mathbf{G}$ . In typical applications, the rank of  $\mathbf{G}_0$  can be significantly lower than  $M$ , and can be increased by adding new reflection paths to the propagation matrices ( $\alpha \geq \beta > 0$ ).

For the reflection free case, the MIMO method performs much better than the initial method. However, the MIMO optimisation is restricted to the target area, outside of which, very high error levels may occur. In the presence of reflections, these errors are mirrored back to the target area, resulting in significant reduction of performance, as can be seen in the figure 5.

Theoretically the optimal solution would be to account for an infinite number of reflections ( $\beta \rightarrow \infty$ ). Figure 6 displays the results of updated MIMO computations, where the first order image sources are taken into account in the PSF optimisation. This corresponds to the  $\beta = 1$  case in equation (9). Comparing the cases of  $C_{1,0}^M$  and  $C_{1,1}^M$ , significant improvement can be seen. However by introducing the second order image sources, the performance radically drops.



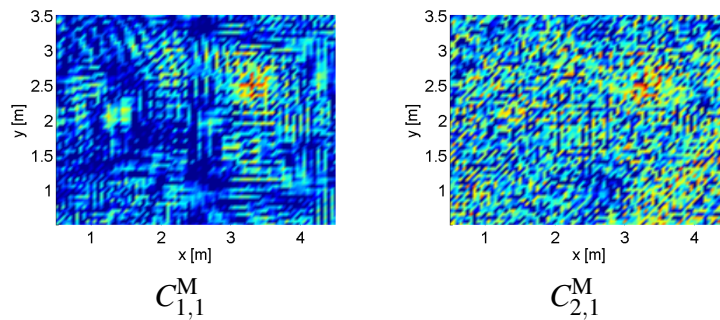


Figure 6: Point spread function  $C_{\alpha,1}^M$  of the random microphone array at the higher frequency case

## 5 A MODIFIED APPROACH

Our modified approach makes use of the known image source distribution. If the room's geometry is a priori known, then we can separately focus on the image sources. We now introduce the following intermediate point spread function:

$$\left[ \mathbf{C}_{\alpha,\beta}^P \right]_{f'_s} = \frac{1}{M} \sum_{m=1}^M \frac{g_{ms}^\alpha}{g_{f'_\beta m}^0} \quad (10)$$

where  $g_{f'_\beta m}^0$  incorporates all the transmission amplifications to the microphone  $m$  from each focus point  $f$  and its separate image sources  $f'$  up to the reflection order  $\beta$ . The resulting point spread matrix is of dimensions  $S \times (RS)$  where  $R$  denotes the number of images up to the order  $\beta$ . The resulting point spread function is obtained by multiplying the intermediate point spread matrix by its pseudo inverse:

$$\left[ \mathbf{C}_{\alpha,\beta}^{PM} \right]_{f_s} = \left[ \mathbf{C}_{\alpha,\beta}^P \right]_{f'_s} \left[ \mathbf{C}_{\alpha,\beta}^P \right]_{f'_s}^+ \quad (11)$$

The advantage of the approach is that the image sources are seen by the microphone array under independent direction characteristics.

## 6 CONCLUSION

In this paper we have demonstrated the effect of the reflections in closed enclosures on the point spread functions of basic beamforming algorithms. The presence of reflections results in the performance drop of these algorithms. We have shown that incorporating first and second order reflection paths into the Green's function does not lead to significant improvement. However by separately focusing on the image sources, a promising modified approach has been proposed. Further work will focus on the usage of three-dimensional, randomly distributed microphone arrays in closed space applications.

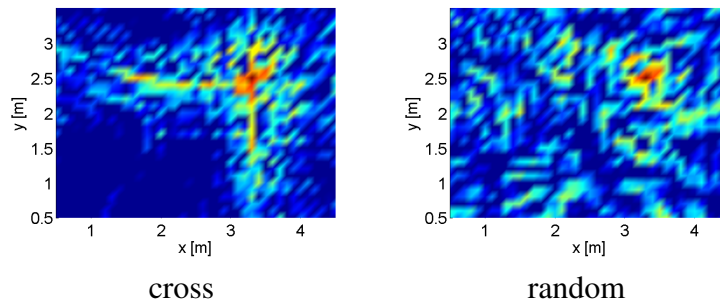


Figure 7: Modified point spread function  $\left[ \mathbf{C}_{1,1}^{\text{PM}} \right]_{f_s}$  of the cross-shaped and random microphone array at the higher frequency case

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