POTENTIALITY OF TIME-REVERSED ARRAY PROCESSING FOR LOCALIZING ACOUSTIC SOURCES IN FLOWS

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ABSTRACT

The present study aims at assessing the potentiality of the time-reversal (TR) technique to localize sound sources in flows, in the context of wind-tunnel measurements. The proposed method is a two-step procedure : the pressure fluctuations are recorded on a linear array of microphones located outside the flow; then the back-propagation of the acoustic waves is simulated by using the time-reversed Linearized Euler Equations (LEE), with the experimental signals as input data, in order to estimate virtually the position of the sound source from which the waves were radiated. The advantage of the proposed method is that the use of the LEE permits an exact description of the convection and refraction of the radiated waves during their propagation to the array. An experiment in a wind-tunnel has been set up, in which a monopolar sine-wave source is located in the flow. The TR results indicate that the source position can be estimated only if the source-array distance is known. However, further simulations of the experiment show that realistic experimental configurations are possible, for which the source position can be accurately estimated without this knowledge.

1 INTRODUCTION

Since the pioneering works of Fink [5], the Time-Reversal (TR) technique has been extensively developed over the last two decades, and is now widely used in many fields of acoustics as medical imaging, geophysics, underwater acoustics...The principle of the TR technique is the following : the acoustic pressure radiated by a source is spatially sampled by a set of transducers. Then the acquired signals are time-reversed, and re-emitted from the array, so that the acoustic wave is back-propagated in the medium. This technique allows to focus on the original sound source, even when the propagation medium is heterogeneous. In the context of the propagation of sound in flows, the propagation medium can be seen as a heterogeneous medium, due to the spatial variations of mean flow, generating convection and refraction of the waves propagating from the source to the microphone array. The TR technique is then a promising technique for performing the localization of sound sources in flows.

In the context of beamforming measurements in aeroacoustics, the flow effects on propagation are usually taken into account by performing posterior corrections using analytical models [1], and by using simplified flow profiles [6]. Conversely, the TR technique could model exactly the wave propagation through any arbitrary mean flow profiles, provided that the used time-reversed equations can model adequately the wave propagation in the flow.

Nevertheless, very few applications in aeroacoustics and audible range are reported in the literature, due to the complexity of the required set up. In ultrasonic applications transducers can receive and emit signals whereas audible frequencies need microphones and loudspeakers. It is why in aeroacoustics and audible range applications, numerical codes are preferred for simulating the acoustic back-propagation. To our knowledge, only a very recent application of the TR technique to aeroacoustics has been proposed in the literature [4]. In this study, Deneuve et al. present numerical TR results based on the Euler equations. On one hand, they studied numerically a vibrating surface whilst on the other hand, the shear layer noise and used complex differentiation to tag acoustic disturbances. In the present study, the linearized Euler equations are used to model the sound propagation through the flow. The goal is to apply the time-reversed simulation by using experimental data, in order to localize a monopolar sound source located in a flow by using a microphone array located outside the flow.

In Section 2, the experimental set-up is presented. A monopolar sound source is located in an anechoic wind-tunnel facility, and the acoustic field is recorded over a uniform linear array. Some hot-wire measurements are carried out to characterize the mean flow velocity profile. This experimental profile is implemented in the numerical solver based on the linearized Euler equations. In section 3, the numerical solver used in this study is described. In section 4, the time-reversal process of the equations is presented. An application of the proposed technique, based on experimental data, is carried out. Some further results, based exclusively on simulations are then presented, in order to demonstrate the potentiality of the TR technique in the context of wind-tunnel measurements.

2 Experimental set up

An experimental device has been developed to validate the procedure for source localization by using the time-reversal technique. The investigations were carried out in the anechoic Eiffel-type wind tunnel EOLE of the PPRIME Institute in Poitiers (Figure 1). The nozzle exit has a square cross-section of $(0.46 \times 0.46)\text{m}^2$, and the test section has a length of 1.32m. The wind tunnel has a maximum operating wind speed of 40m/s ($M = 0.12$), which was the wind speed used during the present investigation. The loudspeaker is embedded in a flat and rigid surface that links the exit of the nozzle to the collector at 0.43m downstream of the nozzle exit. The loudspeaker has a reasonably flat response in the frequency range [250-15000] Hz. Sound measurements were made with an uniform linear array of 17 Brüel&Kjaer equidistant phased microphones, model 4957. In this paper, the distance between two successive microphones is fixed to $d = \lambda/2$ (with λ the acoustic wavelength). The array, fixed on a three-dimensional moving system, allows for careful positioning relative to the source position. The array is placed outside the flow, parallel to the mean flow at a vertical distance fixed to H=1.41m. The array center (the ninth microphone) is then placed vertically above the source and can be moved in the upstream and downstream directions. The calibration of the array was performed without any flow. In the following, x and y denote the streamwise and the vertical directions respectively.

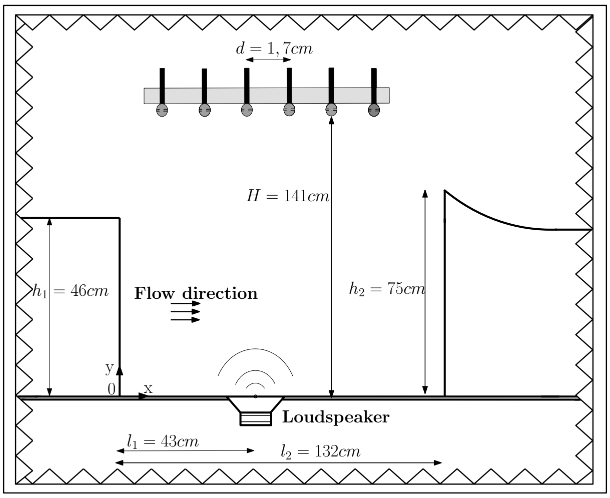

Figure 1: Schematic of the experimental set up.

In this work, pure-tone source signals are used. The output frequency of the loudspeaker was set to 10kHz, with data sampled at 81920Hz over a 30s period. After the mean component subtraction, the signal is filtered by a finite impulse response bandpass around the source frequency.

Flow measurements are also undertaken: mean velocity profiles are characterized by hot-wire measurements. Each profile is made of 78 measurement points over 0.7m. Three positions upstream, three positions downstream and right above the loudspeaker are explored. These measurements are compared with an analytic profile U(x,y) proposed by Candel et al. [3] :

$$U(x,y) = \frac{U_0}{2}\left(1 - \tanh\left(\frac{2(y-y_c)}{\delta}\right)\right), \tag{1}$$

where δ is the shear layer thickness and y_c is the inflexion point. Figure 2 shows the profiles of the analytic model compared to experimental data. The profiles show a good agreement with the analytical model except for low velocities in the shear flow region, due to hot-wire measurement limitations, and for the boundary layer. In the following, the boundary layer effects are neglected. This model for the velocity profile is acceptable for simulating the acoustic propagation in the wind tunnel by using a linearized Euler equations solver.

3 Numerical solver : Linearized Euler Equations (LEE)

In this work, the numerical solver used to simulate the acoustic propagation by taking into account the mean flow effects is based on the solving of the LEE. The linear interactions between

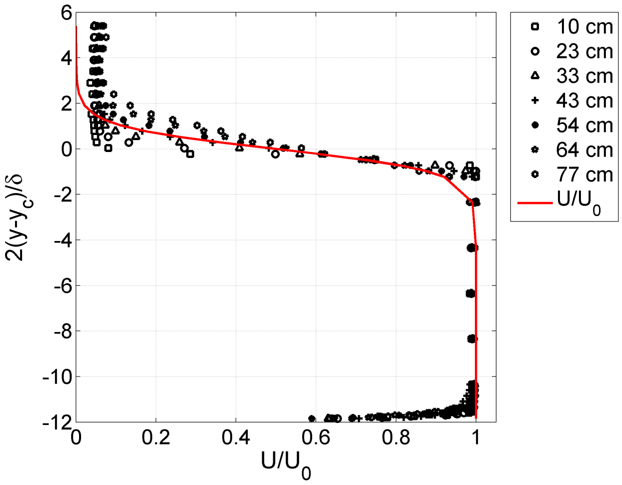

Figure 2: Comparison between experimental and analytical mean velocity profiles. 43*cm is located directly above the loudspeaker position,* 10*,* 23 *and* 33 *cm are located upstream, and* 54*,* 64 *and* 77 *cm are downstream.*

the acoustic field and the mean flow are known to be well described by this model [2]. The linear system of first order equations is obtained by decomposing the density, the velocity components and pressure (ρ, u, v, p) into mean and fluctuating quantities, denoted respectively by the index $_0$ and the symbol $'$. The LEE can be written in bidimensional case as

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \mathbf{S} = 0, \tag{2}$$

with

$$\mathbf{U} = \begin{pmatrix} \rho' \\ \rho_0 u' \\ \rho_0 v' \\ p' \end{pmatrix} \quad , \quad \mathbf{E} = \begin{pmatrix} \rho_0 u' + \rho' u_0 \\ \rho_0 u' u_0 + p' \\ \rho_0 u_0 v' \\ p' u_0 + \gamma p_0 u' \end{pmatrix} \quad , \quad \mathbf{F} = \begin{pmatrix} \rho_0 v' + \rho' v_0 \\ \rho_0 u' v_0 \\ \rho_0 v_0 v' + p' \\ p' v_0 + \gamma p_0 v' \end{pmatrix} \quad , \tag{3}$$

where **U** is the unknown flux, **E** and **F** are the fluxes which take into account all convection and refraction effects on acoustic propagation. **S** represents the source term.

The space is discretized with a 4^{th} order center Dispersion Relation Preserving scheme (DRP) [7]. A 4^{th} order Runge-Kutta scheme is used for time integration. Non-reflective boundary conditions are used to simulate free acoustic field conditions [7]. The acoustic propagation resulting from a pulsating monopole placed in an uniform flow has been studied as a validation test of the code as proposed in reference [2].

4 Time Reversal (TR)

4.1 Principle of TR

The basic principle of TR is a two-step procedure. Let us consider an acoustic source emitting a wave $p(\vec{r}, t)$ in a lossless fluid medium. In the first step, the acoustic field is recorded on a surface. In the second step, the recorded acoustic pressure is time-reversed ($p(\vec{r},-t)$), and re-emitted from this surface. The generated time-reversed sound field can be shown to focus on the initial source position, which ultimately allows its localization. This principle is explained in detail in reference [5] for an ultrasonic field. The TR technique has already been used in many fields of acoustics, like medical imaging, underwater acoustics, non destructive control...

In aeroacoustics, the LEE are commonly used for propagating acoustic waves in flows and seem to be a powerful tool for applying the TR technique for localizing acoustic source in flows. However, the time invariance of LEE after the TR process must be confirmed beforehand.

The LEE are time-reversed in the following way : t is changed into $-t$, and the time derivative is consequently modified. The new set of equations is then different from the original LEE : the invariance of LEE is not preserved, and the TR process can not be applied. To ensure the invariance of LEE by TR, the velocity directions have to be reversed too. For the time-reversal of LEE, the change of variables can then be expressed as :

- $t \rightarrow -t$,
- $\rho(x,y,t) \rightarrow \rho(x,y,-t)$,
- $u(x,y,t) \rightarrow -u(x,y,-t)$,
- $v(x,y,t) \rightarrow -v(x,y,-t)$,
- $p(x,y,t) \rightarrow p(x,y,-t)$.

In this way the LEE are invariant by TR. Moreover, the reversing of velocity directions involves that the convection and refraction effects are properly accounted for.

The first step of the TR procedure is the recording of fluctuating quantities. During the computation of the direct problem, the variables have to be recorded on, either a surface surrounding the source (this configuration is called the Time Reversal Cavity, noted down TRC in the following), or a part of this surface (configuration called Time Reversal Mirror, TRM in the following). In practice, the TRC is difficult to set up experimentally, because it requires a lot of microphones, however, in the context of numerical simulations, the TRC procedure can be carried out more easily. In the results presented in this study, the recorded pressure data are either experimental (Section 4.2), i.e. recorded in a wind-tunnel facility (presented in Section 2), or numerical (Section 4.3).

The second step is the emission of the reversed quantities by solving numerically the time-reversed LEE. The flow direction is reversed, and the source is suppressed. On the cavity or mirror, each quantity is read from the last time step t_{end} until the first one t_{init}, acting as a source. With the LEE, it means that, at each time step, a new Dirichlet condition is imposed where the cavity or mirror is located. The time-reversal process can be summed up as shown in Figure 3.

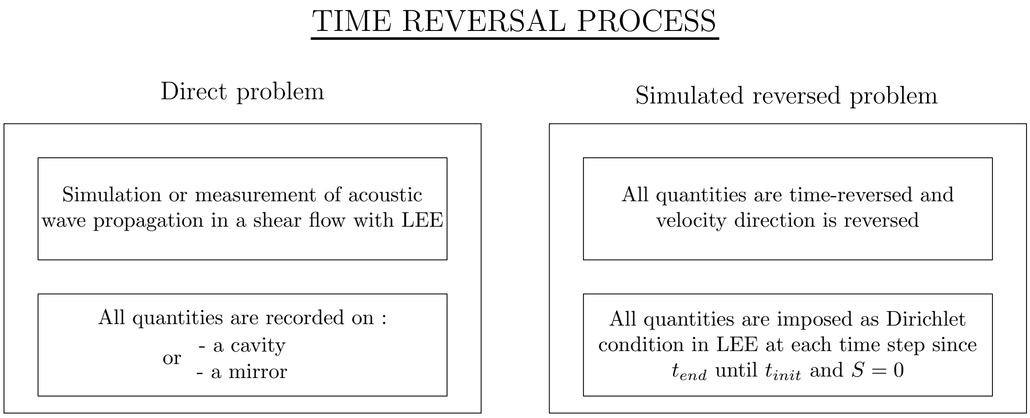

Figure 3: Principle of the time-reversal procedure.

To check the feasibility of the application of time-reversed LEE to the localization of sound sources in flows, the case of an acoustic pulse located in an uniform flow at the center of the numerical domain is studied. The Mach number is equal to $M = 0.25$. All quantities are recorded on the cavity surrounding the source. For the direct problem, an acoustic wave propagates to the boundaries and leaves the computational domain, then all quantities are time-reversed. Figure 4 shows the instantaneous pressure field at three time steps for the direct problem (time step numbers $itime = [0; 170; 270]$) and the reversed problem ($itime = [T - 0; T - 170; T - 270]$, $T = 500$ being the duration of the direct simulation). The wavefront departs from a spherical wavefront, which illustrates the effect of convection on propagation. For the time-reversed problem, the shape of the wavefront looks very similar to the direct problem. The reversed wave back-propagates and converges to the initial pulse position.

However, experimentally, a mirror configuration is easier to set up than a cavity configuration, and the only recorded quantity on a microphone array is the acoustic pressure. Then, in practice, only the reversed pressure and density can be used as a source for the back-propagating simulation. The density fluctuations are simply deduced from the pressure fluctuations by using the assumption of isentropic transformation. So, it is interesting to assess the source localization performance of both TRC and TRM, with all quantities reversed, or only with the pressure and the density reversed.

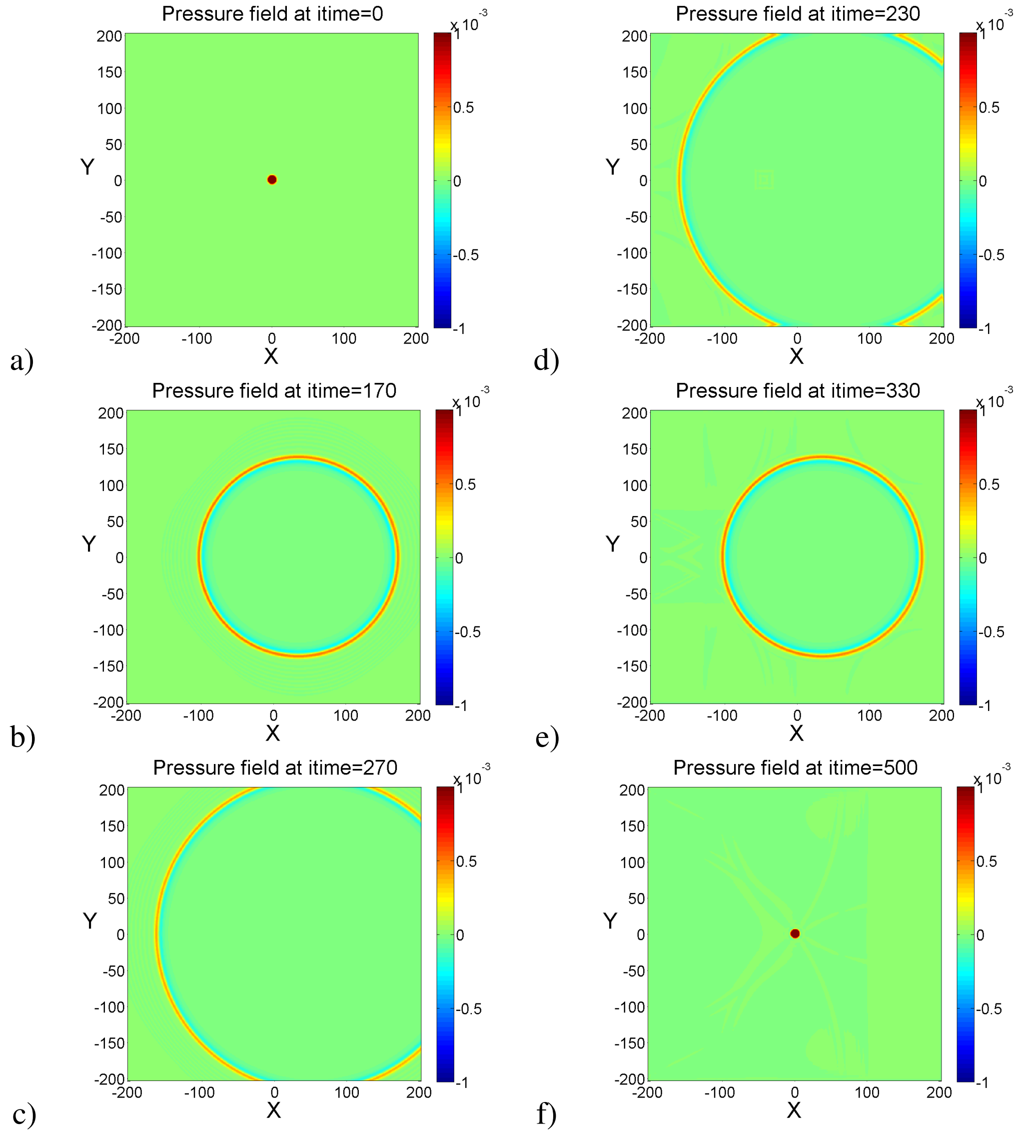

Figure 4: Comparison between direct and time-reversed simulations, for an acoustic pulse in the center of the computationnal domain. Figures a - b - c are three instantaneous pressure fields at three different time-steps and d - e - f are the instantaneous pressure fields at the same reversed time-steps.

Figure 5 shows the pressure profiles along the axis $y = 0$, at t_{end}, of the time-reversed simulation (data of Figure 4.f). The pressure profile at t_{init}, for the direct problem (data of Figure 4.a) is compared with the TRC and TRM with all quantities reversed, and with only p' and ρ' reversed. For each case, the maximum of the pressure profile is located at $x = 0$, indicating that the source position is well localized; however, it is noticeable that the amplitude and shape are different from the initial one ("direct problem" curve). For the TRC with all variables reversed the amplitude is roughly 15% lower. The cavity position is not located on the last point of the computational domain, because the three last points before the domain boundaries are used for the radiation boundary condition. Then a part of the acoustic energy is radiated outside the cavity, which leads to a back-propagated pulse with a lower amplitude. For the TRC (with p' and ρ' only), the two velocity components are not time-reversed and the amplitude is 30 % lower than the original one. Finally the TRM exhibits the lowest amplitude together with a broader shape. This discrepancy can be explained by the fact that acoustic waves are radiated from only one boundary (the mirror). Consequently, the TR technique is shown to localize an acoustic source in a uniform flow. The TRC with all variables reversed is the best case although it is experimentally difficult to set up. In spite of the fact that the original amplitude is not well recovered, the TRM configuration provides a good localization. In the following, the TRM(p',ρ') configuration is used with experimental and numerical data input.

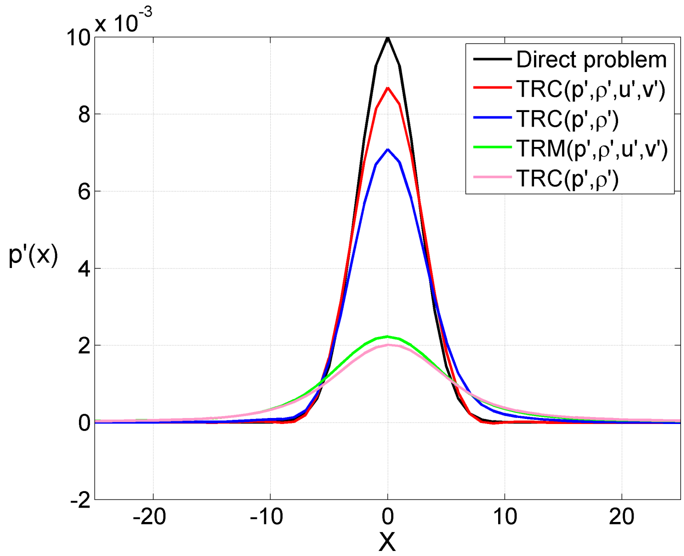

Figure 5: Comparison between the pressure profile of the direct problem at $y = 0$, and the pressure profiles as retrieved by the TR procedure, for the TRC and TRM configurations, and differents sets of time-reversed quantities.

4.2 TR with experimental data input

In this part, an application of TR to the localization of a monopolar acoustic source with experimental data is studied. The experimental set up has been described in Section 2. A loudspeaker emits a stationary sine wave at 10kHz in a uniform shear flow ($M = 0.12$). The wavelength at 10kHz is the reference length and corresponds to eight meshing points in the numerical calculation. The grid size and characteristic distances are set in relation to this reference length. The acoustic pressure is recorded on a microphone array (17 microphones and $d = \lambda/2$) outside the flow. The central microphone of the array is located $25cm$ downstream from the source (i.e., 58 points numerically). The flow velocity profile of Eq. (1) is used for the numerical solver. The microphone array signals are time-reversed and used as input data for the LEE simulation. The source position to be estimated is located at $x_s = 96$ and $y_s = 31$. The microphone array is located at $y = 362$, and the shear layer is located at $y = 138$.

An instantaneous pressure field is plotted in Figure 6. The instantaneous pressure field shows an acoustic beam back-propagating toward the source position. The slight bending of the beam is due to the refraction of the waves entering the flow. One main difference with the pulse case is that the experimental source is permanent (as are most natural aeroacoustical sources). The back-propagated beam passes through the source region but there is no focalization in this area. So the source position in y must be known for this simulation. However, if the source position in y_s is known, the time-averaged squared acoustic pressure can be plotted along a line $y = y_s$, as depicted in Figure 7. A main lobe (together with smaller secondary lobes) is observed, with a maximum located at $x = 96$ matching the position x_s : in this way the source position can be estimated if the distance source-array is previously known. But in some practical cases this parameter is not reachable. This is why, in the following section, this experiment is simulated with LEE in order to find the experimental configuration that could permit to overcome this drawback.

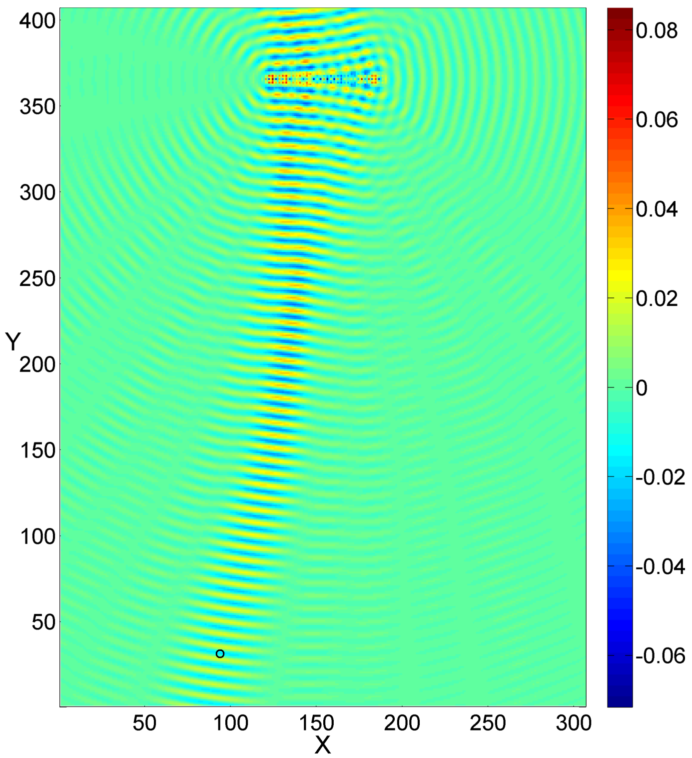

Figure 6: Instantaneous time-reversed pressure field with experimental input data. The black circle indicates the source position.

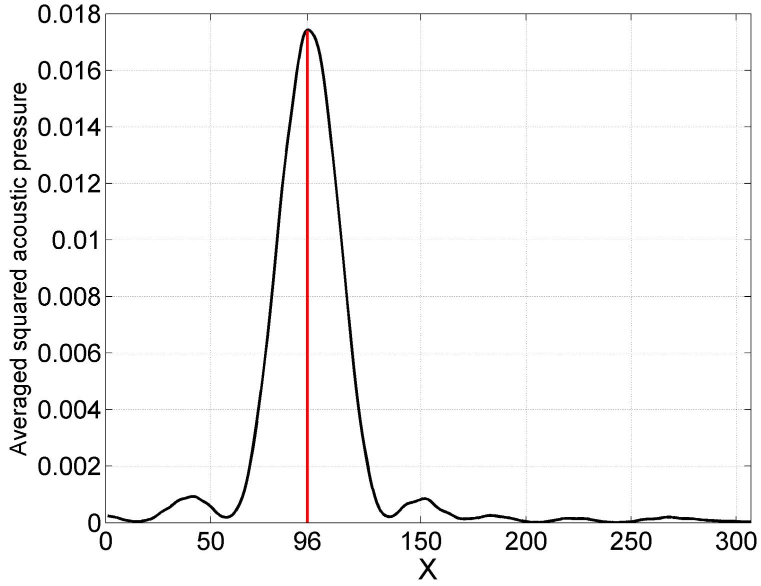

Figure 7: Averaged squared acoustic pressure, $\sum_{t_i} p'^2(x,t_i)$ at $y = y_s$.

4.3 TR with numerical data input

In the next part, TR results are presented, with numerical data input. The simulation of the experimental configuration is computed by using the LEE. In this simulation, the array central microphone is located just above the source. The uniform shear layer is given by Eq. (1), the Mach number is $M = 0.12$. The quantities (p', ρ') are stored on 17 nodes modeling the microphone locations, and the distance between two nodes is half-a-wavelength. Then the TR procedure is carried out : the velocity directions are reversed in the numerical code and the pressure and the density at the microphone locations are time-reversed and used as input data for the LEE solving. Two simulations illustrating two different array lengths and source distances are presented in the following.

The spatial distributions of the time-averaged squared pressure (as computed by the TR procedure) for two different source-array distances ($H_1 = 331$, $L_1 = 64$ and $H_2 = 170$, $L_2 = 176$), are depicted respectively in Figures 8 and 9. The first case illustrates a configuration where H is much larger than the array length, and the second one is a case where H is approximately equal to the array length. In both cases, the microphone array emits an acoustic beam towards the source position. In the following, the region of the back-propagated beam where the time averaged squared pressure is maximum is called the focalization spot, and is marked with a red circle. In the case (H_1, L_1) (Figure 8), the source position can be found only if y_s is known, as in the previous section. However, the focalization spot is located very far from the actual source position, marked with a black circle. Conversely, in the case (H_2, L_2) (Figure 9), the source position and the position of the focalization spot match together very well, indicating that a total localization of the source is possible, both in x and y, without any *a priori* knowledge of the y position of the source.

These preliminary results seem to indicate that increasing the array length, together with decreasing the source-array distance, enables a satisfying focalization onto the source position. It is likely that in such a favorable case, the array intercepts a portion of the wavefronts that is sufficiently curved to allow a satisfying focalization onto the source in terms of distance. Current works aim at having a more general understanding of the optimal conditions for setting up the TR experiment in the wind-tunnel measurement context.

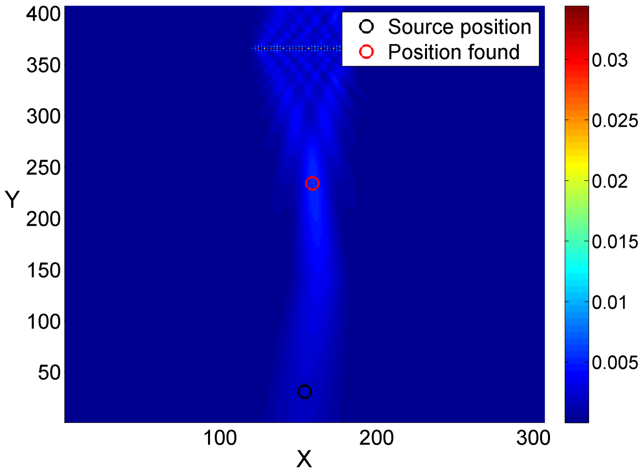

Figure 8: Time-averaged squared pressure for the TR simulation, for the configuration ($H_1 = 331$, $L_1 = 64$). The black circle indicates the source position, and the red circle indicates the maximum indicated by the focalization spot.

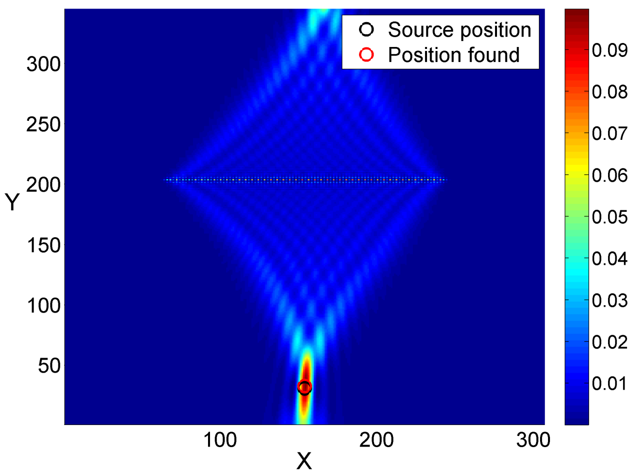

Figure 9: Time-averaged squared pressure for the TR simulation, for the configuration ($H_2 = 170$, $L_2 = 176$). The black circle indicates the source position, and the red circle indicates the maximum indicated by the focalization spot.

5 CONCLUSIONS

The present study aims at assessing the potentiality of the time-reversal (TR) technique to localize sound sources in flows in the context of wind-tunnel measurements. The proposed method is a two-step procedure : the pressure fluctuations are recorded by a linear array of microphones located outside the flow; then the back-propagation of the acoustic waves is simulated by using the time-reversed Linearized Euler Equations (LEE), with the experimental signals as input data, in order to estimate virtually the position of the sound source from which the waves were radiated.

The required transformations of the LEE that enables the invariance of the equations, has been first presented. The effect of using only pressure and density as input data, and only a linear mirror instead of a cavity have been demonstrated.

An experiment in a wind-tunnel has been set up, in which a monopolar 10kHz sine-wave source is located in the flow. The results, obtained from the acquisition of the pressure waves radiated outside the flow onto an array with 17 microphones array, indicate that the source position can be estimated only if the source-array distance is known. Further simulations of the experiment tend to show that realistic experimental configurations are possible, for which the source position can be estimated without this knowledge. It is likely that the array requires to intercept a portion of the wavefront that is curved enough to allow the estimation of the source distance. The optimal experimental configurations for setting the TR technique in wind-tunnel measurements are currently under investigation.

The advantage of the proposed method is that the use of the LEE permits an exact description of the convection and refraction of the radiated waves during their propagation to the array. This method can then be coupled with a prior velocity characterization of the mean flow, which enables an optimal modelization of the flow effects on propagation, and subsequently, an optimal localization of the sound source.

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