AARC BENCHMARK 1 REVISITED

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ABSTRACT

In this paper, the first benchmark problem of the 2006 AARC Phased Array Workshop is reconsidered. The challenge was the determination of the spectra emitted by two acoustic sources from synthetic time data of a small array of microphones. With this benchmark problem, the merits of two new advanced array processing methods are demonstrated. These methods are CSEM (Cross-Spectral Estimation Method) and CLEAN-SC (CLEAN based on spatial Source Coherence). It was found that both methods are well able to estimate the source spectra. For CSEM it was necessary to use a full CSM (Cross-Spectral Matrix). CLEAN-SC was able to recover the source spectra also when the CSM diagonal was removed.

NOMENCLATURE

AARC	Aero-Acoustics Research Consortium
CB	Conventional Beamforming
CLEAN-SC	CLEAN based on spatial Source Coherence
CSEM	Cross-Spectral Estimation Method
CSM	Cross-Spectral Matrix of microphone array
FFT	Fast Fourier Transform
SEM	Spectral Estimation Method
A	source power
$\mathbf{B}$	cross-spectral matrix of sources
B_{jk}	source cross-power
$\mathbf{C}$	averaged CSM
C_{mn}	microphone cross-power
F	cost function
$\mathbf{G}$	matrix consisting of steering vectors
$\mathbf{g}$	steering vector
j	source index
K	number of sources
k	source index
m	microphone index
N	number of microphones
n	microphone index
$\mathbf{p}$	pressure vector
S	subset of all possible (m,n)-combinations
T	subset of all possible (j,k)-combinations
γ^2	spatial source coherence
φ	loop gain

1 INTRODUCTION

In 2006, the Aero-Acoustics Research Consortium (AARC) organised the “Engine Noise Phased Array Workshop”, which took place 11-12 May 2006, in Cambridge, MA. Part of the workshop was devoted to a discussion of two benchmark array problems. The first benchmark problem was the determination of the spectra emitted by two acoustic sources from synthetic time data of a small array of microphones. The second problem concerned the location of speakers inside a duct, using actual array measurements. At the workshop several researchers presented their methods and estimated solutions.

In this paper, we will reconsider the first benchmark problem (“Benchmark 1”). The reason is that new array processing methods have become available, the benefits of which can be demonstrated with this benchmark problem. The first method is CSEM (Cross-Spectral

Estimation Method). This is an extension of SEM (Spectral Estimation Method), which was developed by Blacodon and Elias [1]. The second method is the deconvolution method CLEAN-SC (CLEAN based on spatial Source Coherence) developed by Sijtsma [2]. This method proved to be successful in estimating absolute source levels, also when the diagonal of the CSM (Cross-Spectral Matrix) is removed. Diagonal removal is often inevitable in wind tunnel array measurements [3].

After a brief description of the benchmark problem, this paper starts with an eigenvalue analysis of the CSM, which yields first estimates of the source spectra. Next, results of CB (Conventional Beamforming) are reported. Then, the results obtained with the advanced beamforming methods SEM, CSEM, and CLEAN-SC are discussed. The source spectra calculated with the above-mentioned methods are compared to the actual source spectra.

2 BENCHMARK PROBLEM

Benchmark Problem No. 1 of the AARC Engine Noise Phased Array Workshop featured a small array of 8 microphones. This array, which was located in the plane $z = 0$, had a lay-out as shown in Fig. 1. Synthetic pressure time data were made available as emitted from two acoustic sources located in (–0.2032, 0.0, 0.762) and (0.2032, 0.0, 0.762). The duration of the time signals was 6 s and the sample rate was 22000 Hz. The challenge was to determine the individual frequency spectra of these two sources.

The results in this paper are based on a CSM, which was obtained by performing FFT's on blocks of 512 samples, with Hanning window and 50% overlap. Consequently, the frequency band width was 42.97 Hz and the number of averages was 515. The average spectrum at the array microphones is shown in Fig. 2.

In Fig. 2 the actual source spectra are shown as well. The levels are scaled to the average array spectrum. These scaled source spectra are also shown as reference in other figures of this paper.

3 EIGENVALUE ANALYSIS

Suppose there are K acoustic sources, each inducing a pressure vector $\mathbf{p}_k$, which is the N-dimensional vector of complex pressure amplitudes at the N microphones. Then the total pressure vector is

$$\mathbf{p} = \sum_{k=1}^{K} \mathbf{p}_k \ , \tag{1}$$

For the CSM we have

$$\mathbf{C} = \mathbf{p}\mathbf{p}^* = \sum_{j=1}^{K} \sum_{k=1}^{K} \mathbf{p}_j \mathbf{p}_k^* \ , \tag{2}$$

where the asterisk means complex conjugate transposition. Thus, the rank of $\mathbf{C}$ does not exceed K. If the sources are mutually incoherent, then the terms with $j \neq k$ vanish through averaging, and the following expression remains:

$$\mathbf{C} = \sum_{k=1}^{K} \mathbf{p}_k \mathbf{p}_k^* . \tag{3}$$

Then, **C** is a matrix whose rank is equal to *K*. In other words, the number of non-zero eigenvalues is equal to the number of incoherent sources. Since **C** is Hermitian (invariant to complex conjugate transposition) and positive definite, its eigenvalues are non-negative and the corresponding eigenvectors form an orthogonal set. The eigenvectors (or "principal components") correspond to virtual sources, which need not coincide with the physical incoherent sources.

For the benchmark problem, the spectra of the four largest eigenvalues of **C** are shown in Fig. 3. Since the level of the 3rd eigenvalue can not be neglected as compared to the 2nd, it can be concluded that the rank of **C** is greater than 2. At first sight this seems odd, as there are only two sources. The reason for its presence is that the signals emitted from the sources do not arrive simultaneously at each microphone. In other words, FFT time blocks of different microphones consist partly of noise data that were emitted from the sources at different times. Increasing the block size will lead to a relatively lower level of the 3rd eigenvalue. However, this will be at the expense of a lower number of averages.

Nevertheless, Fig. 3 shows that the spectra of the first two eigenvalues are a good estimate of the two source spectra.

4 CONVENTIONAL BEAMFORMING

The most straightforward way to process phased array data is the CB technique. This is a frequency-domain method, in which powers *A* of sources in points $\vec{\xi}$ in a scan area are determined as follows. Let *N* be the number of microphones, and **C** the measured $N \times N$-dimensional CSM. Further, let **g** be the *N*-dimensional steering vector, which consists of microphone pressure amplitudes induced by a unit monopole point source in $\vec{\xi}$. If *S* is a subset of all possible (*m*,*n*)-combinations, where *m* and *n* are microphone indices, then the source power *A* can be obtained through minimization of

$$F = \sum_{(m,n)\in S} \left| C_{mn} - A g_m g_n^* \right|^2 . \tag{4}$$

The solution is

$$A = \sum_{(m,n)\in S} g_m^* C_{mn} g_n \Big/ \sum_{(m,n)\in S} \left|g_m\right|^2 \left|g_n\right|^2 . \tag{5}$$

In wind tunnel measurements *S* usually contains all (*m*,*n*)-combinations with $m \neq n$, which means that the diagonal is removed from the CSM.

Typical beamforming images (summed to 1/3 octave bands), obtained using CB without diagonal removal, are shown in Fig. 4. The scan area was in the plane $z = 0.762$. At each frequency band, the dominant source can be recognised. The secondary sources, however, are masked by the large side lobes of the main source. Large side lobes are the consequence of the low number of microphones.

In Fig. 5 and Fig. 6 the reconstructed source spectra are shown, with and without removed CSM diagonal, respectively. It can be seen that the reconstructed spectra follow the actual spectra for those frequencies where the source concerned is dominant. In other cases the reconstructed levels are due to side lobes of the main source. Beamforming with the full CSM gives less distortion than with the diagonal removed.

5 SPECTRAL ESTIMATION

The CB method described in the previous section can be generalised to multiple sources. Suppose there are K sources in points $\vec{\xi}_k$, which are associated with steering vectors $\mathbf{g}_k$. Then the $K \times K$-dimensional source cross-spectral matrix $\mathbf{B}$ can be obtained by minimising

$$F = \sum_{(m,n)\in S} \left| C_{mn} - \left(\mathbf{GBG}^* \right)_{mn} \right|^2 , \tag{6}$$

where $\mathbf{G}$ is an $N \times K$ matrix consisting of steering vectors:

$$\mathbf{G} = \begin{pmatrix} \mathbf{g}_1 & \cdots & \mathbf{g}_K \end{pmatrix} . \tag{7}$$

For the benchmark problem, the dimensions of the matrices $\mathbf{B}$ and $\mathbf{G}$ are 2×2 and 8×2, respectively.

We can write Eq. (6) as

$$F = \sum_{(m,n)\in S} \left| C_{mn} - \sum_{(j,k)\in T} G_{mj} B_{jk} G^*_{nk} \right|^2 , \tag{8}$$

where T is the set of source combinations for which non-zero cross-powers are expected. This minimisation problem can be solved by setting the derivative of F with respect to B_{jk} to zero, for each $(j,k)\in T$. The result is:

$$\sum_{(j_1,k_1)\in T} \left(\sum_{(m,n)\in S} G_{mj_1} G^*_{mj} G^*_{nk_1} G_{nk} \right) B_{j_1 k_1} = \sum_{(m,n)\in S} C_{mn} G^*_{mj} G_{nk} . \tag{9}$$

With all $(j,k)\in T$, Eq. (9) forms a straight matrix equation, where the dimension is the number of elements in T. Solution of Eq. (9) yields the source auto- and cross-powers B_{jk}.

If T consists of diagonal elements (k,k) only, then we have basically the SEM method, as described by Blacodon and Elias [1]. Results of this method are shown in Fig. 7 and Fig. 8, again with full CSM and with the diagonal removed. The results with SEM are much better than with CB. Only at those frequencies where the 3rd eigenvalue is close to the 2nd (see Fig. 3), the method seems to fail. There is not much difference between the SEM results with and without CSM diagonal. Only at the highest frequencies, the full CSM results are significantly better.

Now suppose that the set T consists of all possible source combinations, i.e.,

$$T = \left\{ (j,k);\ j = 1,..,K,\ k = 1,..,K \right\} . \tag{10}$$

In that case the method can be called the Cross-Spectral Estimation Method (CSEM). The results of CSEM, with and without CSM diagonal, are shown in Fig. 9 and Fig. 10, including the reconstructed source cross-spectrum. For the case with the full CSM (Fig. 9) the reconstructed spectra look very good. However, when the diagonal is removed there are still problems around 3000 Hz.

The level of the reconstructed cross-spectrum is remarkably high. This is not due to an insufficient number of averages, or to an insufficiently large FFT block size. From the source auto-power and cross-power levels we can calculate the source coherence by

$$\gamma^2 = \left|B_{12}\right|^2 \big/ B_{11}B_{22} = 10^{\left(2\times\mathrm{SPL}_{12} - \mathrm{SPL}_{11} - \mathrm{SPL}_{22}\right)/10}\,, \tag{11}$$

where SPL_{jk} are the source levels as plotted in Fig. 9 or Fig. 10. The source coherence as derived from CSEM with full CSM is plotted in Fig. 11.

6 CLEAN-SC

The CSEM method, described in the previous section, is basically an inverse deconvolution method, i.e., a matrix equation needs to be solved. A more direct deconvolution approach is CLEAN-SC [2]. This method iteratively removes the part of the source plot which is spatially coherent with the peak source. CLEAN-SC has been proven successful in unmasking secondary sources, also when the diagonal of the CSM is removed.

With the current array of only 8 microphones CLEAN-SC needs to be applied with care, because it often occurs that side lobes have higher levels than the main lobes (see Fig. 4). In those cases, CLEAN-SC doesn't recognise the actual source location, and the method fails. Therefore, the source locations appointed by CLEAN-SC were limited to the two (a priori known) source locations.

Results obtained with CLEAN-SC are shown in Fig. 12 and Fig. 13, using the full CSM and with diagonal removal, respectively. The figures show that CLEAN-SC performs well, also when the CSM diagonal is removed. It was beneficial to use a moderate loop gain φ (see [2]) for this case. Here we used $\varphi = 0.5$.

CLEAN-SC is based on the assumption that acoustic sources are incoherent. For the benchmark problem this is not entirely true (see Fig. 11). Since the results look good, the source coherence was, apparently, sufficiently small.

7 ASSESSMENT OF MOST PROMISING METHODS

The most promising methods discussed in the previous sections are CSEM with full CSM, CLEAN-SC with full CSM, and CLEAN-SC with removed diagonal. The reconstructed source spectra obtained with these methods agree well with the actual source spectra (see Fig. 9, Fig. 12, and Fig. 13). For source 1 and source 2, respectively, Fig. 14 and Fig. 15 show the differences between the reconstructed and the actual spectra. For source 1, CSEM shows the best performance up to 7000 Hz. Between 8000 Hz and 9000 Hz, the best predictions are from CLEAN-SC with removed diagonal. For source 2, there is not much difference in performance between the methods considered.

8 CONCLUSIONS

The first benchmark problem of the 2006 AARC Phased Array Workshop was reconsidered. It appeared that the Spectral Estimation Method (SEM), in which the source auto-spectra are calculated by solving a least squares problem, is not able to recover the full spectra of source powers. If the least squares problem is extended with source cross-spectra, then source power recovery is possible for the entire spectrum. This extended SEM method is called CSEM (Cross-Spectral Estimation Method). CSEM performs less well when the diagonal is removed from the CSM. The deconvolution method CLEAN-SC is also able to estimate the source spectra accurately. In contrast with CSEM, CLEAN-SC performs well with diagonal removal too.

ACKNOWLEDGEMENT

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[2] P. Sijtsma, “CLEAN based on spatial source coherence”, *International Journal of Aeroacoustics* 6 (4), 357-374, 2007.

[3] T.J. Mueller (Ed.), *Aeroacoustic Measurements*, Springer, 2002.

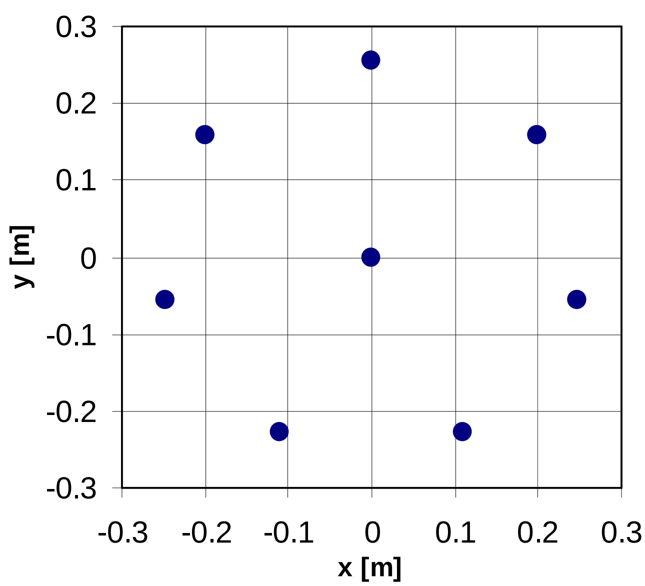

Fig. 1. Microphone lay-out

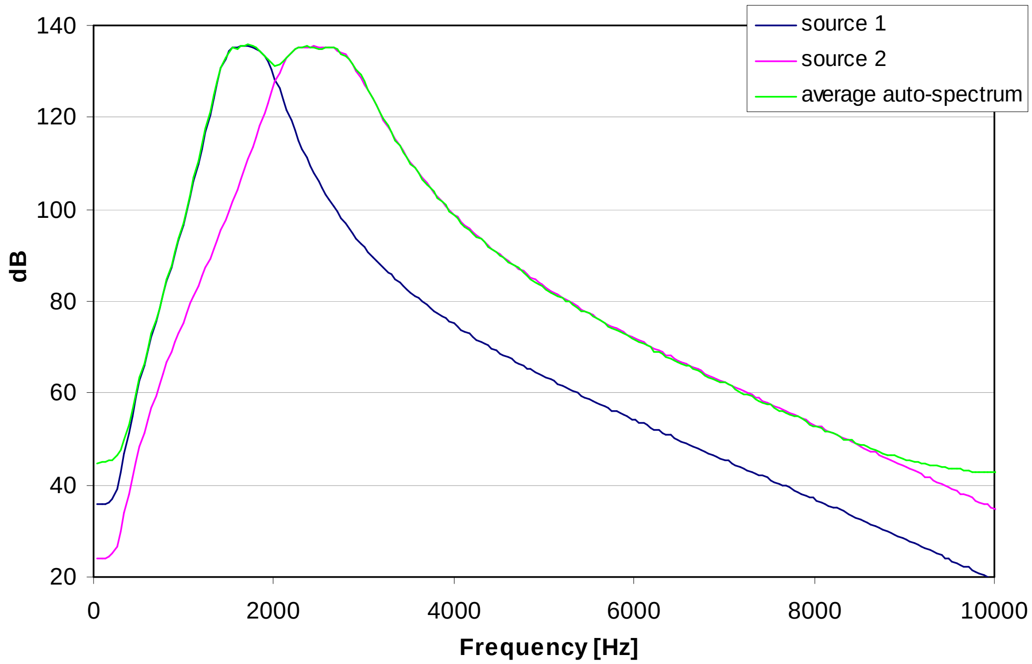

Fig. 2. Source spectra and average spectrum at the array microphones

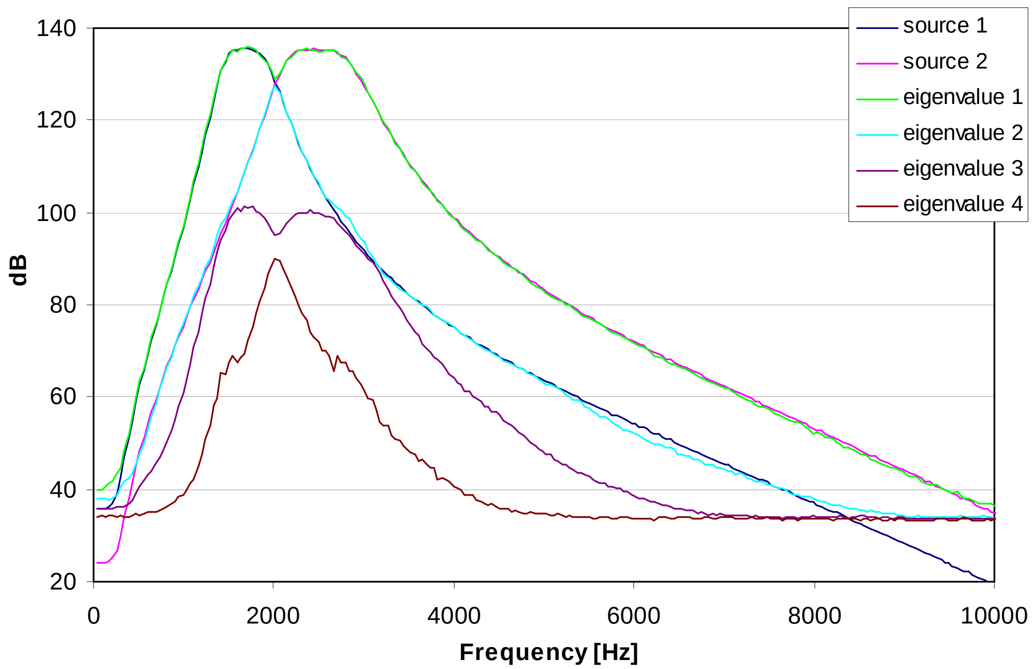

Fig. 3. Spectra of the four largest eigenvalues

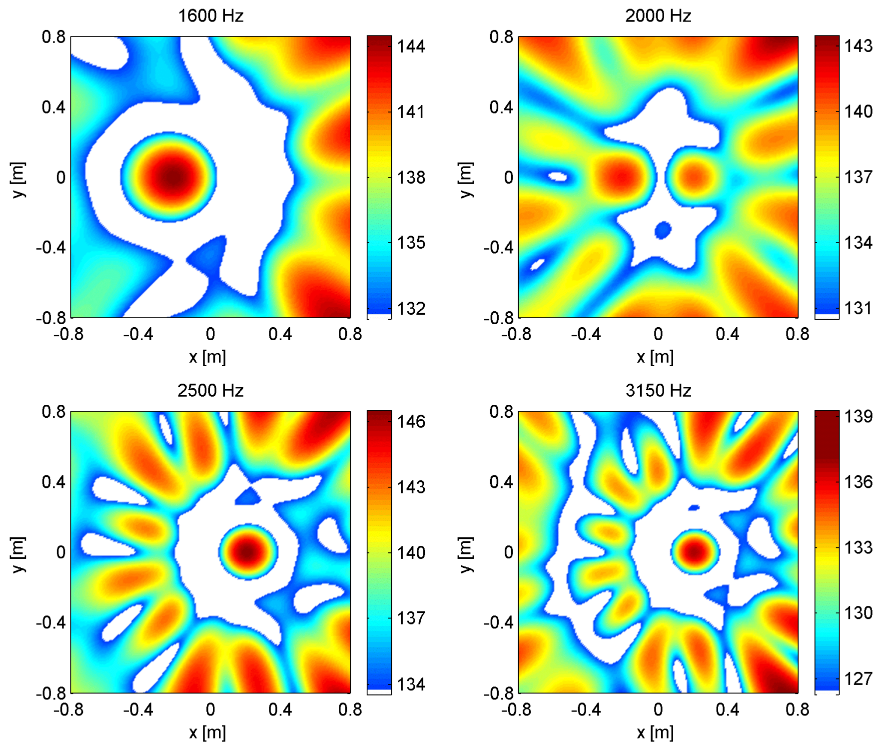

Fig. 4. Typical beamforming images, obtained using CB with full CSM

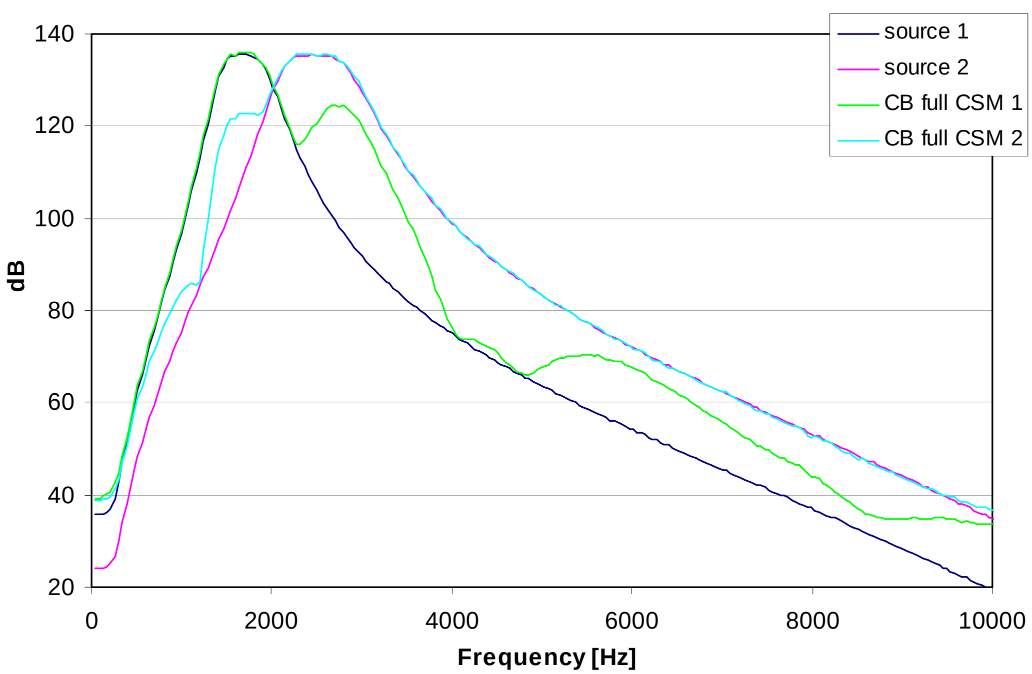

Fig. 5. Reconstructed source spectra using CB with full CSM

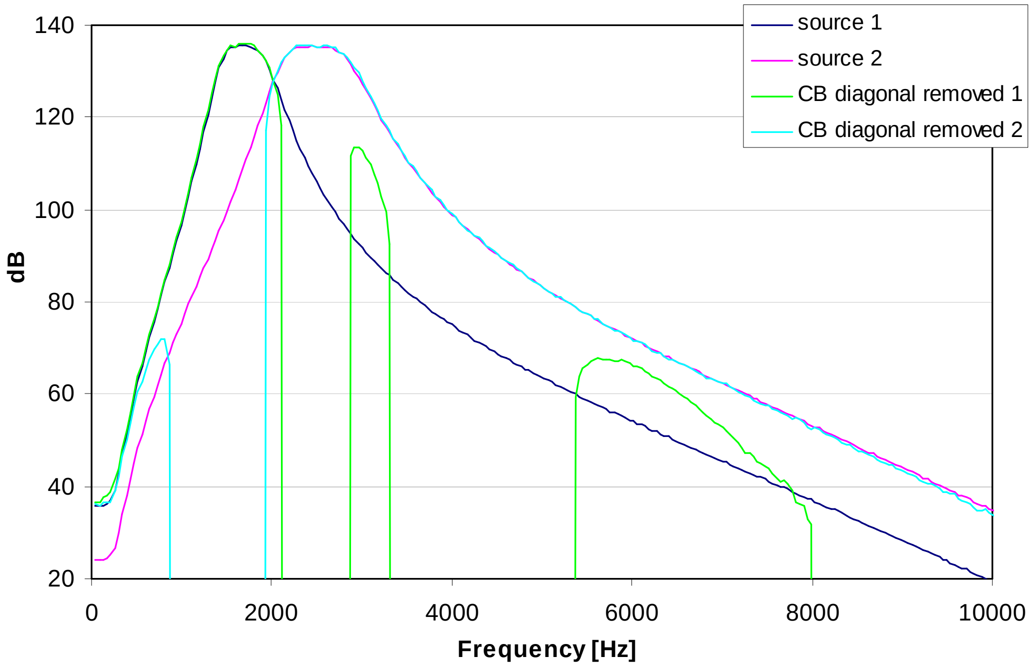

Fig. 6. Reconstructed source spectra using CB with removed diagonal

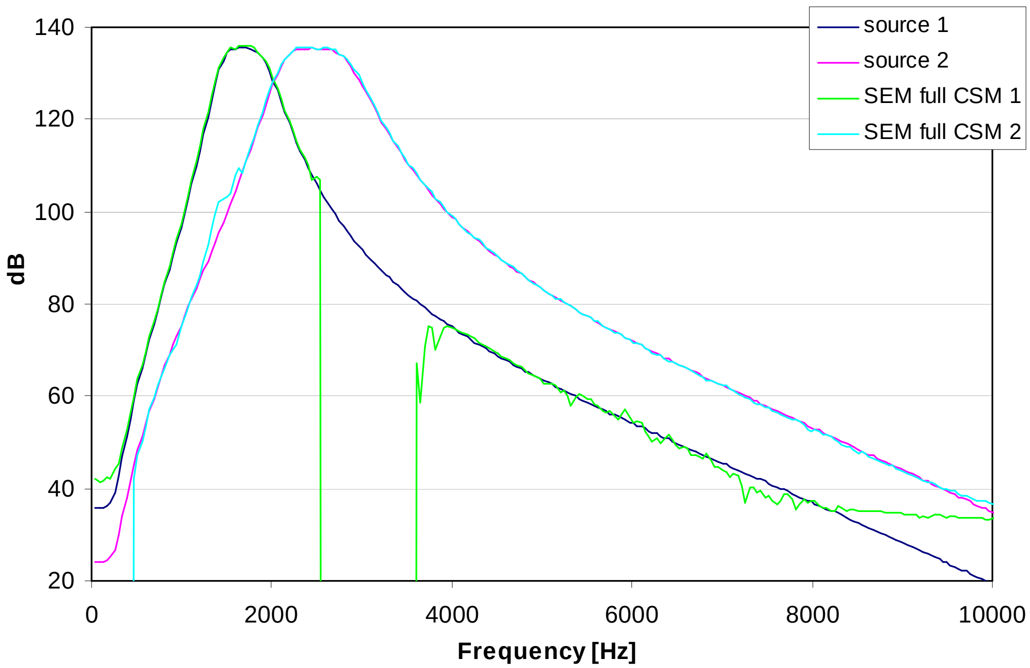

Fig. 7. Reconstructed source spectra using SEM with full CSM

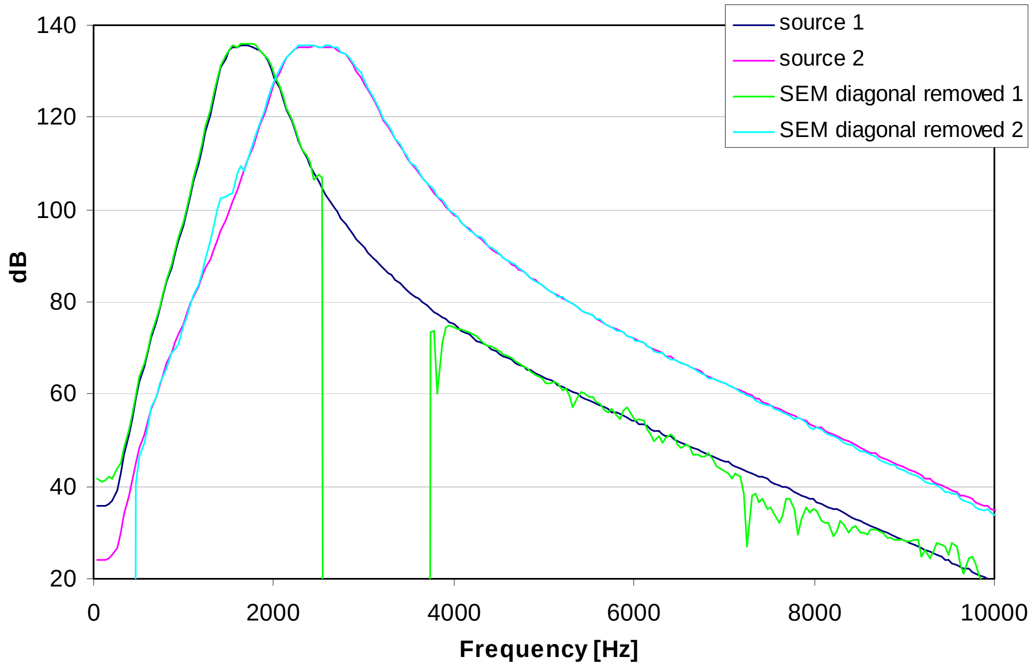

Fig. 8. Reconstructed source spectra using SEM with removed diagonal

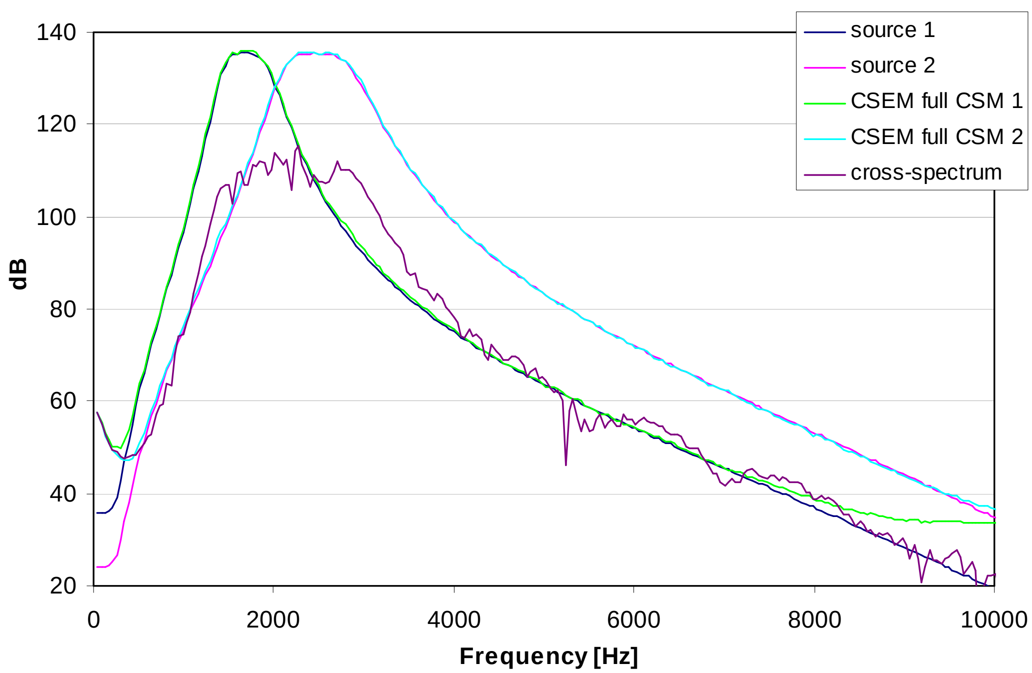

Fig. 9. Reconstructed source spectra using CSEM with full CSM

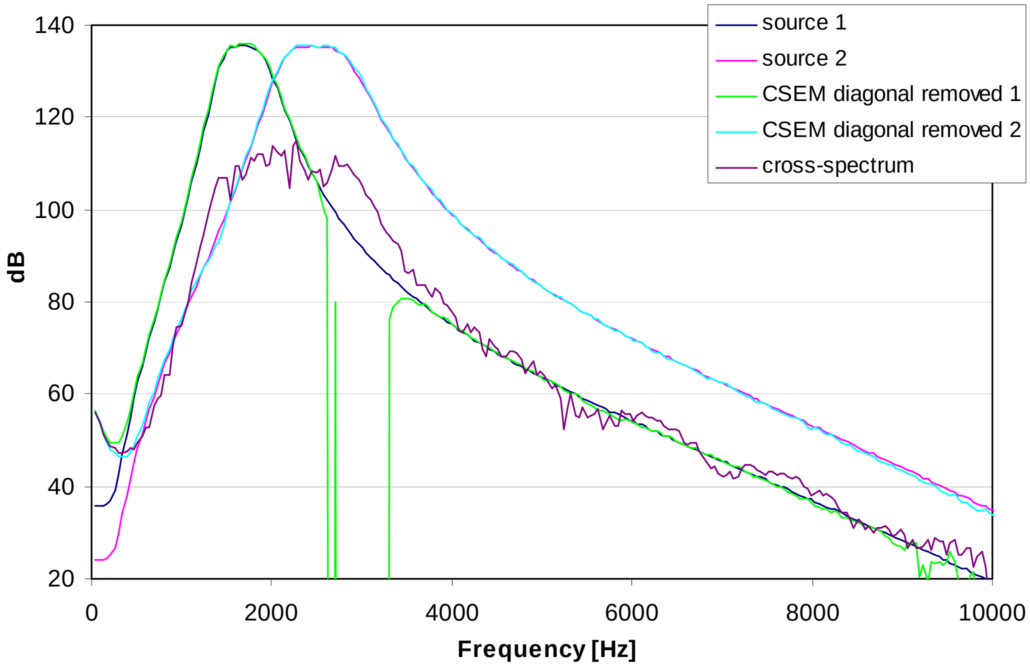

Fig. 10. Reconstructed source spectra using CSEM with removed diagonal

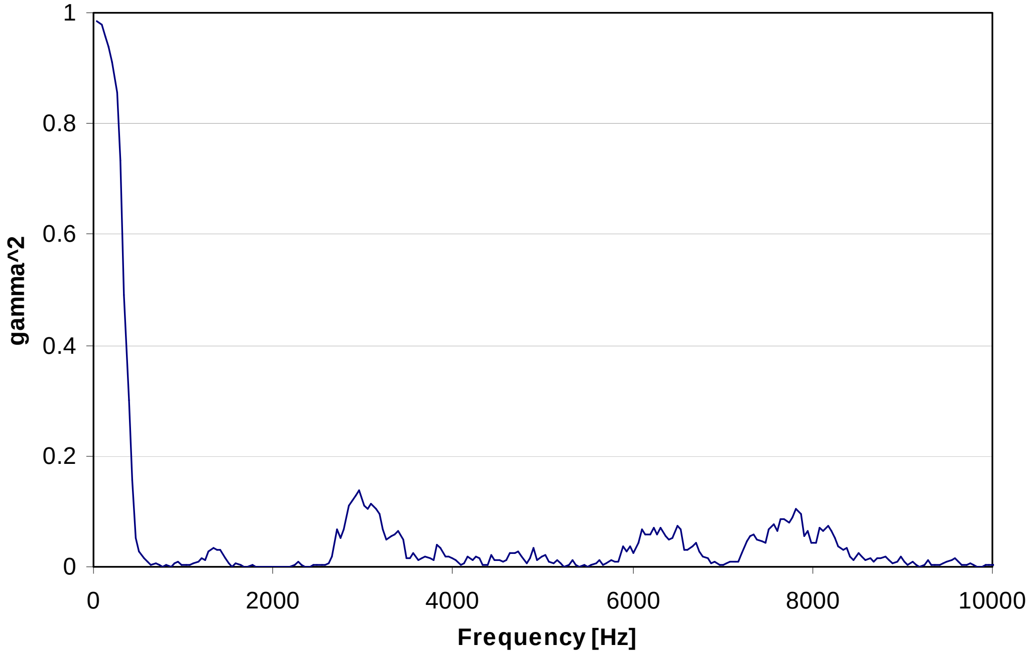

Fig. 11. Coherence between both sources, as derived from CSEM with full CSM

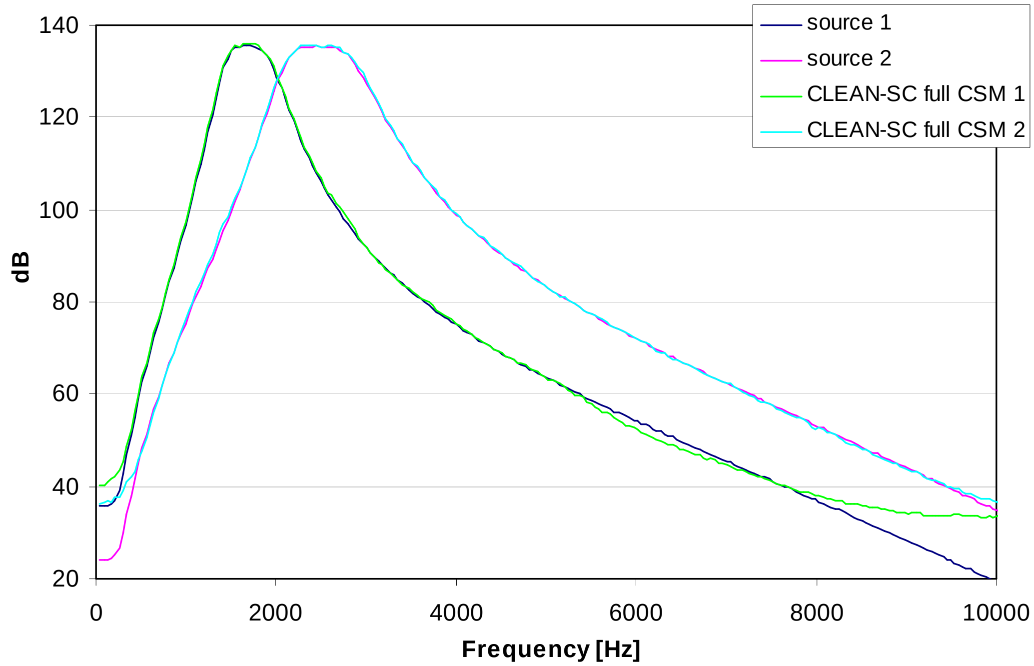

Fig. 12. Reconstructed source spectra using CLEAN-SC with full CSM

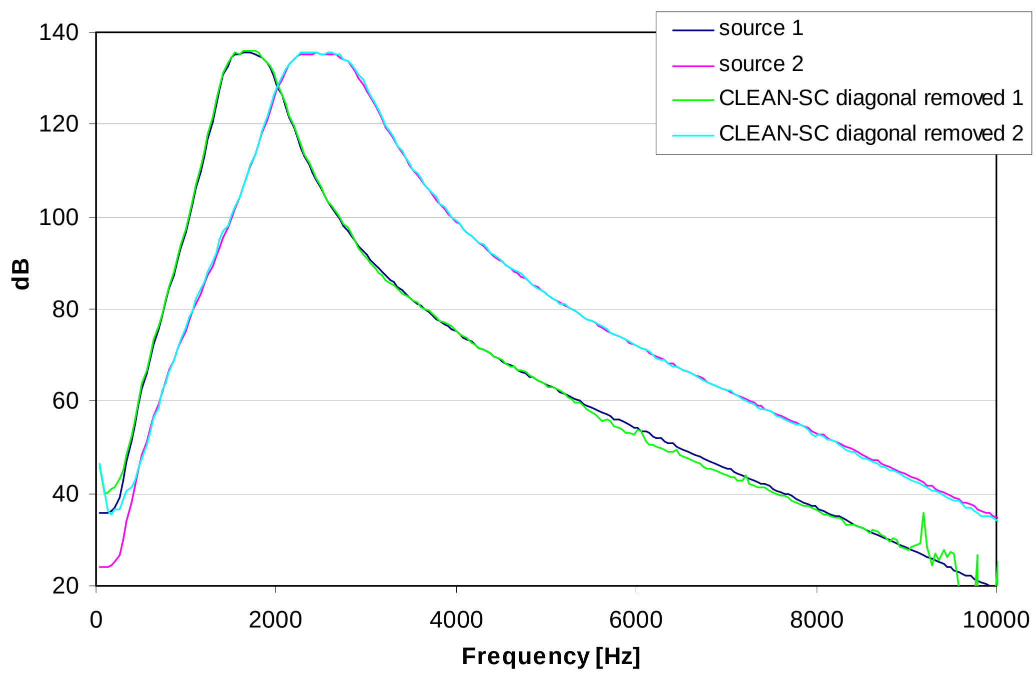

Fig. 13. Reconstructed source spectra using CLEAN-SC with removed diagonal

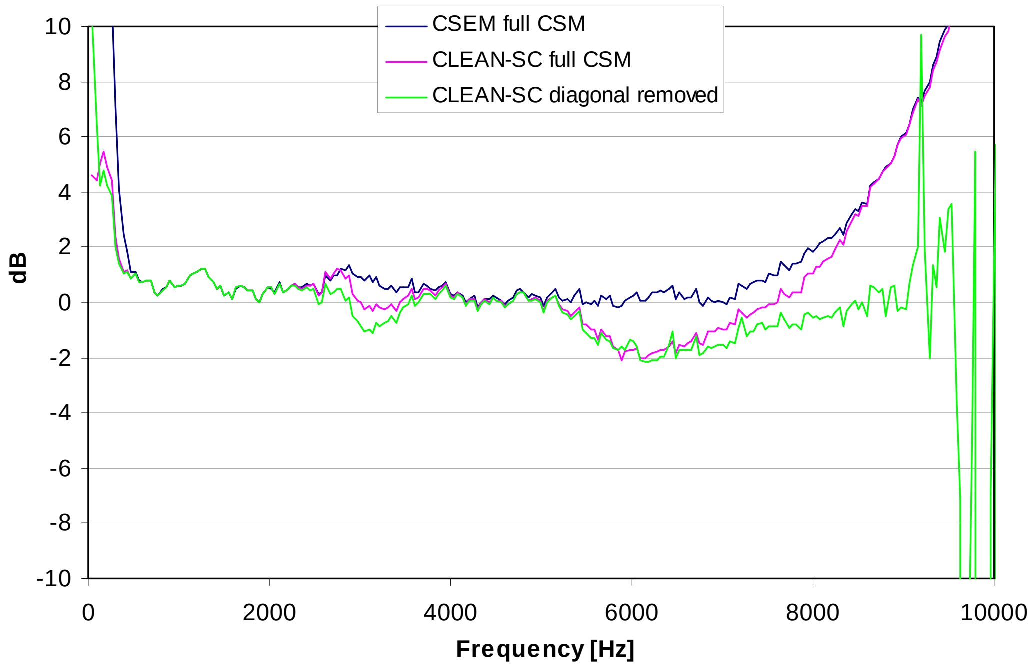

Fig. 14. Difference between reconstructed spectra and actual spectrum of source 1

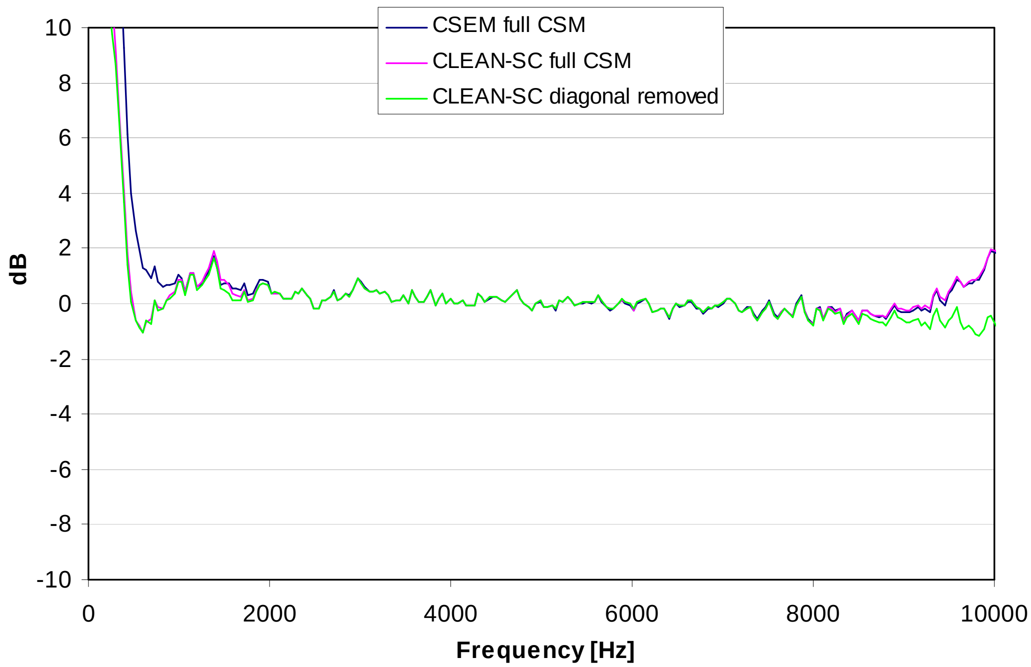

Fig. 15. Difference between reconstructed spectra and actual spectrum of source 2