Quantitative source spectra from acoustic array measurements

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ABSTRACT

While phased microphone arrays are often applied with the focus on acoustic source localization, in many cases quantitative source information is also needed. The estimation of quantitative source spectra from array measurements is not straightforward and a number of methods exist that either use integration of the beamforming map or solve inverse problems for source estimation. An alternative, computationally efficient method is proposed that is based on a signal subspace approach. A short overview of the theory is given and the method is demonstrated using two examples, a four loudspeaker set-up and trailing edge noise measurement.

1 Introduction

When compared to the use of a single microphone, the main advantage of acoustic measurements with phased arrays is that individual sources can be separated from one another and from the always present background noise. Therefore, phased arrays become increasingly popular for the analysis of complex acoustic situations. Many applications of acoustic array measurements aim at the localization of sound sources, either on machinery and vehicles or on laboratory set-ups in the wind tunnel. Nevertheless, in the context of machinery or vehicle noise control as well as in the context of scientific wind tunnel experiments the estimation of reliable quantitative information about the sources is important, too.

The usual processing of array microphone output signals leads to a map of sound pressure contributions. As long as only a single point source is present, the quantitative estimation of the source spectrum from the map is straightforward. The peak level in the map corresponds to that of the source and thus can be used as a quantitative source spectrum. If several sources are present and/or the sources are not compact, but consist of spatially extended source domains, this is no longer true. One popular approach that was successfully applied is an integration technique. Integration is performed over a source region of interest in the map. But, with the use of conventional beamforming techniques the beamforming map shows the source strength convolved with an array-dependent point spread function. This can be taken into account by integrating over a weighted beamforming map [1]. Each point in that beamforming map is weighted by an individual factor that accounts for the integrated point spread function in that point. In a number of computationally less expensive approaches that factor is approximated by a representative factor that does not depend on the location in the beamforming map [1, 2, 3, 4, 5]. Even with rigorous consideration of the point spread function, results from the integration may be influenced by contributions from outside the integration area. All integration approaches require the prior estimation of a full beamforming map.

Another class of methods use inverse approaches for the estimation of quantitative source spectra. The so-called spectral estimation method [6] is a parametric method. The difference between the actual cross spectral matrix of microphone signals and a theoretical cross spectral matrix is minimized. The theoretical matrix is simulated on the basis of contributions from previously defined source areas. Thus, the minimum is achieved by iteratively adjusting the source strengths. The results are quantitative source spectra, but are not unique and depend strongly on the chosen distribution of source areas. The deconvolution approach for the mapping of acoustic sources (DAMAS) [7] is also an inverse approach but aims at the estimation of a "'true"' map of source strengths. The problem is represented by a huge system of equations with as many unknowns as points in the map grid. A result can be obtained by iterative solution, but requires a large number of iterations.

In the present paper a different method is presented. The rationale behind its development were the need for a computationally efficient method that allows the fast processing of data from a large number (some thousand) of single measurements. Thus, the method does not rely on integration nor inverse iterative estimation, but is based on a signal subspace approach that utilizes a very simple model of the sound field. A short overview of the theory is given and the method is demonstrated using two examples.

2 Theory

Consider a microphone array with N microphones. In practice, the sound pressure present at each of these microphones is a superposition of contributions from a number of sound sources and source mechanisms and from additional noise. For M sources, the sound pressures at the microphones are thus given by

$$\mathbf{p} = \mathbf{A}\mathbf{q} + \mathbf{n}, \tag{1}$$

where the sound pressures are the N elements of the vector $\mathbf{p}$ and the source strengths measured as a reference sound pressure are the M elements of $\mathbf{q}$. The $N \times M$ matrix $\mathbf{A}$ contains the respective transfer functions as its elements. Noise that is present at the microphones, but unrelated to any of the sound sources, is represented by the elements of $\mathbf{n}$. The signal q_j of any sound source is assumed to be uncorrelated to any noise signal n_i. Under the additional assumption that all noise signals are of equal amplitude n and are mutual uncorrelated, the cross spectral matrix $\mathbf{G}$ of the microphone signal is given by

$$\mathbf{G} = \mathbf{A}\mathbf{S}\mathbf{A}^H + n^2\mathbf{I} \tag{2}$$

where the superscript H indicates the Hermitian transpose, $\mathbf{S}$ denotes the cross spectral matrix of the source signals and $\mathbf{I}$ is the identity matrix.

The problem of estimating quantitative source spectra is equivalent to the estimation of $\mathbf{S}$ in (2). From measured time histories of sound pressure, however, only an estimation $\hat{\mathbf{G}}$ of the cross spectral matrix is available. The matrix of transfer functions $\mathbf{A}$ is not known a priori. Thus, it is impossible to solve (2) for $\mathbf{S}$ without more information.

Under the assumption that only a single source is present, the problem may be reformulated. The matrix $\mathbf{A}$ reduces to vector $\mathbf{a}$. Introducing a so called steering vector $\mathbf{h}$ with $\mathbf{h}^H\mathbf{a}(\mathbf{x}_s) = 1$, the autospectrum of the source can be estimated by

$$S = \mathbf{h}^H(\mathbf{G} - n^2\mathbf{I})\mathbf{h} \;\text{ or }\; \hat{S} = \mathbf{h}^H\hat{\mathbf{G}}\mathbf{h}. \tag{3}$$

A common choice for $\mathbf{h}$ is $\mathbf{a}(\mathbf{x}_s)/\big(\mathbf{a}(\mathbf{x}_s)^H\mathbf{a}(\mathbf{x}_s)\big)$, where it is usually assumed that $\mathbf{a}(\mathbf{x}_s)^H\mathbf{a}(\mathbf{x}_s) = N$. Using this formulation for $\mathbf{h}$, the influence of possible additional sources on this estimation is minimized, provided that they are evenly distributed over all possible source locations. It can be noted in passing that the removal of the main diagonal from $\hat{\mathbf{G}}$ in (3) also removes any influence of the uncorrelated noise on the estimation.

The above choice for the steering vector in (3) reveals the physical meaning of the estimation. If r_0 is the root mean square distance of all N possible source - microphone distances, then $\sqrt{S}$ is an estimation of the sound pressure in that distance from the source.

In a practical scenario more than one source is usually present. If it is assumed that the signals of all sources are mutual uncorrelated, then the cross spectral matrix $\mathbf{S}$ is a diagonal matrix and thus $\mathbf{G}$ may also be represented by a sum of cross spectral matrices per source [8]. On the other hand side, the cross spectral matrix of the microphone signals is Hermitian and positive definite. Thus, it has only positive eigenvalues. The eigenvalue decomposition of this matrix is

$$\mathbf{G} = \mathbf{V}\mathbf{L}\mathbf{V}^H, \tag{4}$$

where $\mathbf{V}$ is the matrix of the orthonormal eigenvectors and $\mathbf{L}$ is a diagonal matrix populated with the eigenvalues. Following Su and Morf [9], it is reasonable to assume that M of these eigenvalues are large and the remaining, representing the noise, are small and approximately equal, this becomes

$$\mathbf{G} = \mathbf{V}\begin{bmatrix}\mathbf{L}_M + n^2\mathbf{I} & 0\\ 0 & n^2\mathbf{I}\end{bmatrix}\mathbf{V}^H = \mathbf{V}\mathbf{L}_M\mathbf{V}^H + n^2\mathbf{I}, \tag{5}$$

where $\mathbf{L}_M$ is a $M \times M$ diagonal matrix. By comparison with (2) it is obvious that this matrix and the source cross spectral density matrix are similar. From the properties of $\mathbf{a}$ it follows that

$$\mathbf{L}_{M,ii} = N\mathbf{S}_{ii}. \tag{6}$$

Because the entries of $\mathbf{L}$ correspond to that of $\mathbf{S}$, it is not necessary to perform the beamforming algorithm (3) to get quantitative information about the individual source amplitudes. To assign these amplitudes to sources at a location of interest, a reduced, so called orthogonal beamformer may be applied, where $\mathbf{G}$ is replaced by the reconstructed estimation of the cross spectral matrix due to the i-th source $\mathbf{V}_i\mathbf{L}_{M,ii}\mathbf{V}_i^H$. A map of sound pressure contributions estimated using this beamformer will show only contributions from one source. Thus, the map has one distinct maximum that corresponds to the location of the source. This enables the estimation of quantitative source spectra using the following algorithm:

- define the local region of interest,
- calculate $\mathbf{L}_M$ for each frequency,
- estimate position of maximum in orthogonal beamforming map for each diagonal element of $\mathbf{L}_M$,
- if maximum inside region of interest, add appropriate element of $\mathbf{L}_M$ to result.

The overall result corresponds to the combined power density of the sourced within the region of interest. It can be noted that the main computational cost is the estimation of the orthogonal beamforming maps. In practical situations only few of the diagonal elements of $\mathbf{L}_M$ need to be considered. Thus, the computational cost for the method is only a fraction of that necessary for the estimation of a full beamforming map in conventional beamforming.

3 Example results

The method explained above shall be illustrated with two examples. The first example is a test case with four loudspeakers used as sound sources. An array measurement shall reveal the source spectra of all four loudspeakers. The loudspeakers were placed in a quadrangle arrangement with a spacing of 10 cm between them and they were driven with four uncorrelated white noise signals of similar amplitude. The distance between the loudspeakers and the array was 88.5 cm and the array had an aperture of 1.40 m with the 32 microphones arranged in two circles (see also Figure 3).

This test case has some interesting features. The sound sources are closely spaced and of equal amplitude. Thus, at least for lower frequencies beamforming provides no good separation nor reliable quantitative information. Figure 1 shows the conventional beamforming map (CBF-M) at a frequency high enough to extract source locations. Along with this map four orthogonal beamforming maps are shown for the four largest diagonal elements of $\mathbf{L}_M$. The maximum level of the apparent sources in this map differs from that in the CBF-M, but the sum of all four maps gives the same result as the CBF-M. The method explained above was applied and source spectra of all four loudspeakers were extracted. To this end four quadratic regions (20 cm $\times$ 20 cm) or sectors were defined in the beamforming map around the point (-0.01,0) m. Each of these sectors contained one loudspeaker. An example result for the lower left sector is given in Figure 2. It is compared to another measurement where only the loudspeaker in that sector was active. Similar results were obtained for the other loudspeakers and also for pairs of loudspeakers driven with coherent noise. These pairs were recognized by the method as one spatially distributed source.

The second example demonstrates the use of the method for airfoil trailing edge noise. A SD 7003 airfoil with a chord length of 235 mm, a maximum camber of 20 mm and a wingspan of 400 mm was used for the measurements in the aeroacoustic wind tunnel at technical university of Cottbus (BTU). This facility is an open jet wind tunnel especially designed for very quiet

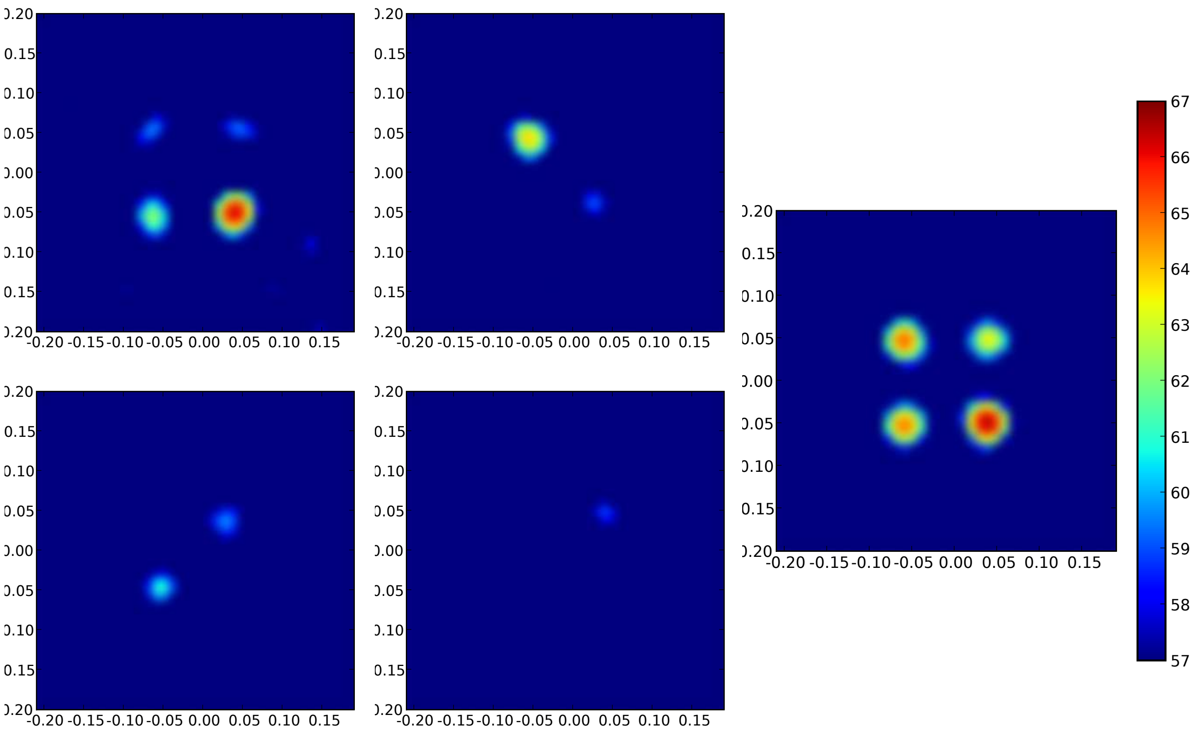

Figure 1: Four loudspeaker assembly at a frequency of 8 kHz: orthogonal beamforming maps (left) and conventional beamforming map (right), sound pressure level in dB

flow. With the specific circular nozzle used for the tests, the jet diameter is 20 cm and the maximum flow speed is 60 m/s. The airfoil was placed in front of the nozzle. Figure 3 shows the set-up that was used as well as the trailing edge source region considered for the estimation of quantitative source spectra. The measurements were carried out at a large number of different configurations and only a small subset of the results can be shown here. Figure 4 shows the third-octave spectrum of the trailing edge noise for three different flow speeds, while in Figure 5 scaled results from a larger number of measurements are plotted over the Strouhal number. The reasonable trend that can be recognized in this last figure shows that the results from the method at different frequencies fit together.

4 Summary

A method for the estimation of quantitative source spectra from acoustic array measurements was presented in this paper. The theory of the method bases on the orthogonal beamforming technique. While beamforming is used for the source localisation, the actual source spectrum is estimated from the decomposed cross spectral matrix. One of the advantages of this approach is the very low computational cost. Using two examples, the sucessful application of the method was demonstrated.

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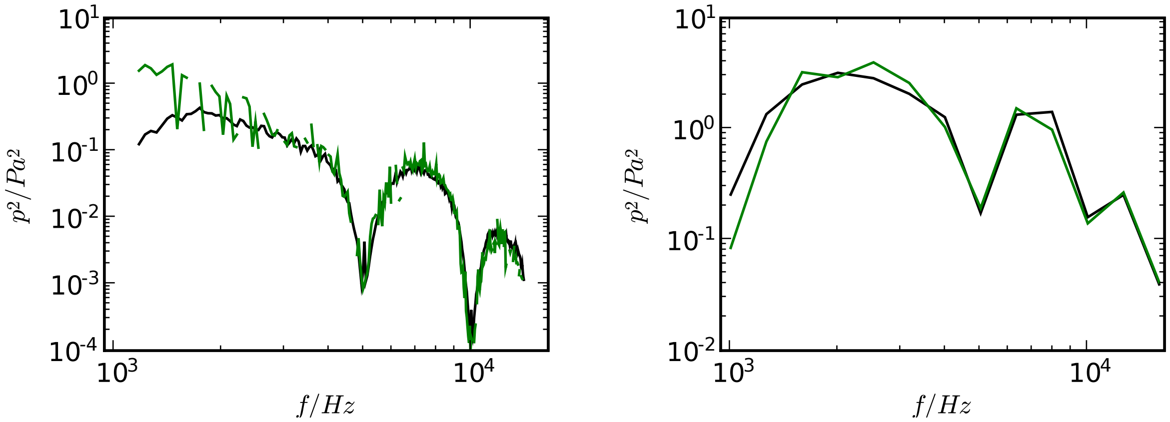

Figure 2: Example results for source spectra from lower left sector — from measurement with four loudspeakers — from measurement with single loudspeaker; auto power spectrum with Δf =46.875 Hz (left) third-octave spectrum (right)

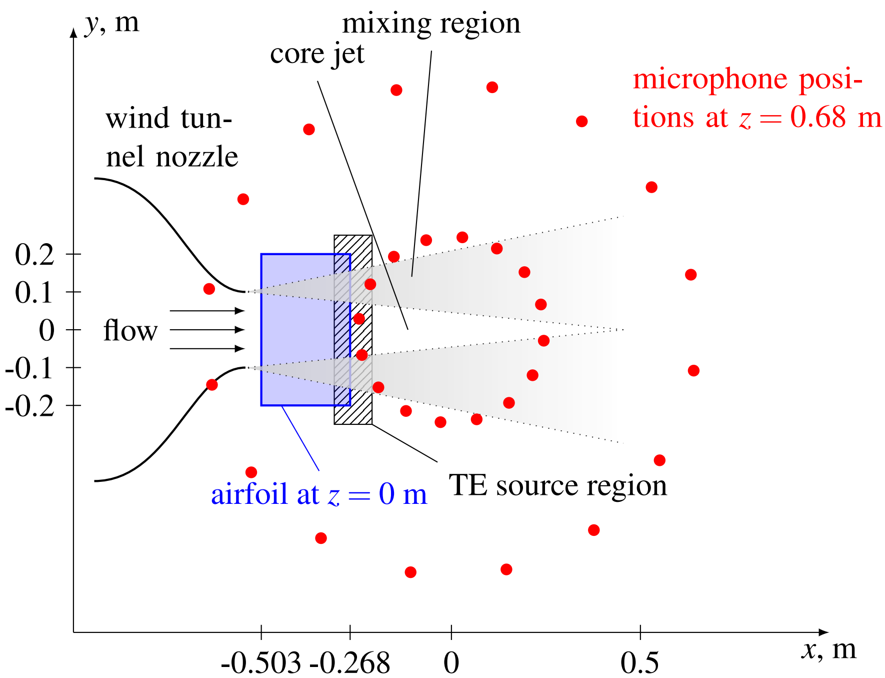

Figure 3: Schematic of the test set-up in the BTU aeroacoustic wind tunnel

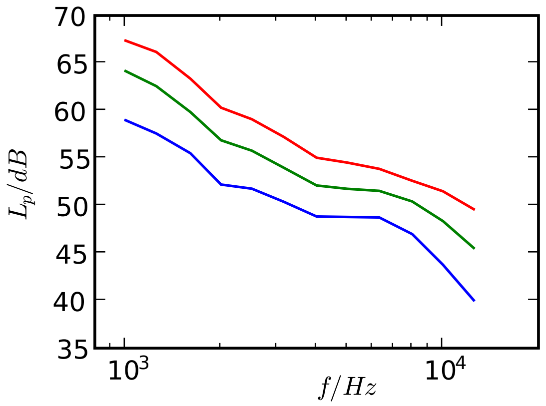

Figure 4: Results for TE noise: third octave sound pressure levels for 0° angle of attack and flow speed U = —— 38.4 m/s, —— 44.3 m/s, —— 49,6 m/s

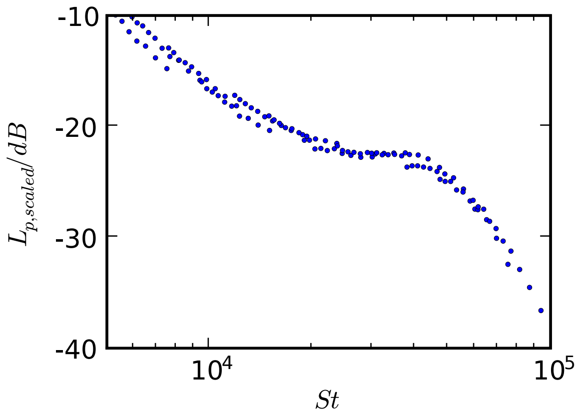

Figure 5: Results for TE noise: scaled third octave sound pressure levels $10\lg(p^2/U^{4.5})$ *for 0° angle of attack as a function of Strouhal number St (based on chord length)*

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